The Operational Logic of Planning for Industrialization

by

MD. ANISUR RAHMAN*

INTRODUCTION

It is wellknown that real national income of an export-oriented primary-producing country may be significantly affected by a change in its terms of trade with the rest of the world. The demand for a country's exports is beyond its control; but a central authority may be able to influence its terms of trade by controlling the other blade of the scissor. Through direct and indirect controls over the allocation of domestic investment, the central authority may seek to regulate the domestic production of export goods and also of import substitutes, thereby influencing the supply of exports and hence terms of trade of the country. To do this rationally is one of the crucial planning problems faced by the majority of primary-producing underdeveloped countries many of which do have some central authority, planning and controlling in one way or the other the direction of investible resources between different sectors of the economy.

The present paper is a study on the optimum allocation of investment in such a country between the primary sector and the industrial sector with a view to maximizing the growth of *real national income.* Real national income is so defined here as to measure the export component of national output in terms of the *real volume of imports that exports command* at the current terms of trade. This is best done by deflating the current value of exports by an index of import prices, or equivalently by inflating exports, valued at *constant base-period prices,* by an index of the terms of trade[1]. The domestically absorbed component of national output on the other hand is expressed in real terms by

---

* The author is Reader in Economics, University of Dacca. He is indebted to Mr. Mati Lal Pal, Staff Economist at the Pakistan Institute of Development Economics, Karachi, for pointing out a flaw in an argument in an earlier draft of the paper, and also to Professor Nurul Islam and other colleagues in the Department of Economics, University of Dacca for opportunities of discussing the work with them. The responsibility for any inadequacies of the paper rests with the author alone.

[1] *Cf.*, J. B. D. Derkson, "International Comparisons of Real National Income: An International Survey", *Income and Wealth*, Series 1. (Cambridge: Bowes and Bowes). This practice is actually being followed in some countries like Pakistan and Palestine. *See*:

i) Central Statistical Office, *Pakistan Statistical Yearbook, 1957.* (Karachi: Central Statistical Office, Government of Pakistan), p. 201.

ii) P. J. Loftus, *National Income of Palestine, 1945.* (Jerusalem: 1948).

using prices of the chosen *base-period* as weights given to outputs of the period concerned[2].

The allocation problem presented in this study is seen mainly from the point of view of operational planning[3]. This means not only that the relevant relationships are assumed to be simple (*e.g.*, linear) so as to be manageable by the type of planning apparatus available in the countries concerned, but also that only those possibilities are considered which may reasonably be *anticipated* by planners of these countries, leaving out other possibilities which may be conceivable but are unlikely to be anticipated. Rational planning, to be positive, must of necessity line up with this point of view.

The study aims mainly in deriving some *qualitative directives* for social planning; and for simplicity of exposition, some abstractions, like ignoring transport costs and other trade barriers and ignoring the existence of other sectors (*e.g.*, service sector) in the economy, are made with the feeling that such 'trimming' would not distort the moral that is being sought. For the purpose of quantitative planning in any specific country, these abstractions can be done away with and the proper structure set up without any departure from the general approach outlined in this study[4].

The major finding of the study is that notwithstanding a possible comparative advantage over the rest of the world in the production of primary commodities, rational planning for a primary-producing country must aim at industrial expansion sooner or later in order to maximize the combined gain in real income from production and trade. It should be sooner, other things given, the greater is the size of investment planned, *i.e.*, the faster is the rate of growth of real national income that is intended. Simultaneous expansion of the primary sector should be planned if the product of this sector is a 'normal' commodity in the country in an aggregative sense; but an eventual contraction of this sector is required if its product is an 'inferior' commodity.

II: THE PROBLEM STATED

A country is predominantly a producer of primary goods, to be called for convenience agricultural goods. Some industrial goods may or may not be

[2] Alternatively, current value of output could be deflated by appropriate domestic price-indices. The two methods do not in general yield the same measure. Both the methods are in practice and there are arguments both in favour and against. In the present study, the method of weighting by base-period prices is followed for the specific reason that it makes analytical computation easier.

[3] It is also a study in *equilibrium theory*, if social, and not merely individual, planning is accommodated in such theory.

[4] For quantitative planning, the structure of course has to be *identifiable* in the econometric sense. The simple relationships assumed in this study, intended as they are mainly for a qualitative analysis, ignore the problem of identification.

produced. A significant part of this country's agricultural product is exported and industrial goods are imported. A central authority, controlling the allocation of investment between the two sectors, has estimates of demand and supply functions for the country's exports, and also of the country's production-possibility function[5] for agricultural and industrial goods. The problem, as it is before the central authority, is to allocate the estimated investible resources available to the economy in some initial period, to be called the base period (identified by the time-subscript $t = 0$), so as to maximize planned real national income in the following period[6] (to be called the plan period $t = 1$).

Real national income in period t ($t = 0, 1$), to be called Z_t, is defined as:

$Z_t = P_o X_t + Y_t + (P_t - P_o). E_t$ where

X_t and Y_t are measures in the index-number sense of the volumes of agricultural and industrial outputs respectively and E_t measures the volume of (agricultural) exports. The industrial good is chosen as the *numeraire* so that its price is always unity. P_t is defined as the terms of trade (both external and internal, abstracting from all factors that might cause a discrepancy) in period t, giving the price of agricultural good in terms of industrial good. Real national income as we have defined it is the sum of: *a*) national output domestically absorbed, measured at constant base-period prices; and *b*) exports measured in terms of the real volume of imports they command, *i.e.*, current value of exports deflated by the index of import prices. Since the index of import prices is always unity by our choice of *numeraire*, we have

$$Z_t = P_o (X_t - E_t) + Y_t + P_t .E_t$$
$$= P_o X_t + Y_t + (P_t - P_o). E_t .$$

The structure of the economy is defined by the following linear estimates:

Supply function for exports: $S_{E_t} = aX_t + bY_t + cP_t + h$

Demand function for exports: $D_{E_t} = dP_t + q$

Production-possibility function: $kx + y = S$, where

S_E, D_E are amounts of exports supplied and demanded respectively; x, y are *changes* in agricultural and industrial outputs respectively as between the plan period and the base period, *i.e.*, $x = X_1 - X^o$ and $y = Y_1 - Y_o$; S is a measure of the amount of investible resources

[5] The familiar term in economics for a function showing the maximum amount of either good that can be produced in combination with different given amounts of the other good, corresponding to a *given* total amount of available resources.

[6] A multiperiod extension is made later in Section VII.

where z gives the change in real income over the two periods concerned. Z_o being known, Z_1 will be maximized when z is maximized.

Similar operations on the export supply and demand functions yield

$$S_{E_1} = ax+by+cp+(aX_o+bY_o+cP_o+h)$$

and

$$D_E = dp+(dP_o+q).$$

Assume that in the base period, excess demand for exports is zero[8] at the base-period terms of trade P_o. Then

$$E_o = aX_o+bY_o+cP_o+h = dP_o+q$$

and the balance requirement $S_{E_1} = D_{E_1}$ for the plan period can be written simply as:

$$ax+by+cp = dp, \text{ whence}$$

$$p = \frac{ax+by}{d-c}$$

This yields

$$E_1 = E_o+dp = E_o+\frac{d(ax+by)}{d-c}, \text{ so that}$$

$$p.E_1 = \frac{E_o(ax+by)}{d-c}+\frac{d(ax+by)^2}{(d-c)^2}$$

$$= \frac{(ax+by)P_o}{e_d-e_s} + \frac{d(a^2x^2+b^2y^2+2abxy)}{(d-c)^2}, \text{ where}$$

e_d = *price elasticity of demand for exports at* P_o, and

e_s = price elasticity (a partial-derivative concept) of supply of exports at P_o, so that

$$e_d = d\cdot\frac{P_o}{E_o} \text{ and } e_s = c\cdot\frac{P_o}{E_o}$$

Hence

$$z = P_ox+y+p.E_1$$

$$= P_ox+y+\frac{P_o(ax+by)}{e_d-e_s}+\frac{d(a^2x^2+b^2y^2+2abxy)}{(d-c)^2}$$

$$= w_{11}x^2+w_{22}y^2+w_{12}xy+w_1x+w_2y, \text{ where}$$

$$w_{11} = a^2d; \quad w_{22} = b^2d; \quad w_{12} = 2abd;$$

$$w_1 = P_o\left(1+\frac{a}{e_d-e_s}\right); \; w_2 = 1+\frac{P_ob}{e_d-e_s}$$

---

[8] On the basis of the linear estimates postulated. This assumption also is released later in Section V.

We need, then, to maximize the expression

$z = w_{11}x^2 + w_{22}y^2 + w_{12}xy + w_1x + w_2y$, subject to the constraint

$kx + y = S$.

Define $F = z + \lambda g$ to give the constraint maximization function with $g = kx + y - S = 0$ and λ the familiar lagrange multiplier. The first-order condition for maximization is then given, in terms of partial derivatives, by

1) $z_x/z_y = g_x/g_y$ and *2*) $kx + y - S = 0$

1) gives

$$\frac{2w_{11}x + w_{12}y + w_1}{2w_{22}y + w_{12}x + w_2} = k,$$

or $2w_{11}x + w_{12}y + w_1 = k(2w_{22}y + w_{12}x + w_2)$

or $(2w_{11} - kw_{12})x + (w_{12} - 2kw_{22})y = kw_2 - w_1$

or, on substituting terms,

$$\frac{1}{(d-c)^2}\left[(2a^2d - 2kabd)x + (2abd - 2kb^2d)y\right]$$
$$= k\left(1 + \frac{bP_o}{e_d - e_s}\right) - P_o\left(1 + \frac{a}{e_d - e_s}\right)$$

or, on simplification,

$ax + by = T$, where

$$T = \frac{(d-c)^2}{2d(a-kb)}\left[k - \frac{P_o(a-kb)}{e_d - e_s} - P_o\right]$$
$$= \frac{(d-c)^2}{2d(a-kb)}\left[k + \frac{P_o(a-kb)}{e_s - e_d} - P_o\right]$$
$$= \frac{(d-c)^2}{2d(a-kb)}\left[R - P_o\right], \quad \text{where } R = k + \frac{P_o(a-kb)}{e_s - e_d}.$$

The second-order condition for maximization is given by

$$\Delta_1 = \begin{vmatrix} F_{11} & F_{12} & g_1 \\ F_{21} & F_{22} & g_2 \\ g_1 & g_2 & 0 \end{vmatrix} > 0; \text{ and } \Delta_2 = \begin{vmatrix} F_{22} & g_2 \\ g_2 & 0 \end{vmatrix} < 0$$

$$\Delta_1 > 0 \text{ gives } \begin{vmatrix} 2w_{11} & w_{12} & k \\ w_{12} & 2w_{22} & 1 \\ k & 1 & 0 \end{vmatrix} > 0$$

$$\text{or } 2w_{11}(-1)-w_{12}(-k)+k(w_{12}-2kw_{22}) > 0$$

$$\text{or } 2(kw_{12}-k^2w_{22}-w_{11}) > 0$$

$$\text{or } \frac{2}{(d-c)^2}\cdot(k\cdot 2abd-k^2b^2d-a^2d) > 0$$

$$\text{or } -\frac{2d}{(d-c)^2}(a-kb)^2 > 0 \quad \text{, which is satisfied if } d < 0;$$

$$\text{and } \Delta_2 < 0 \text{ gives } \begin{vmatrix} 2w_{22} & 1 \\ 1 & 0 \end{vmatrix} < 0, \text{ which is necessarily true.}$$

It would be difficult to expect, in a linear estimate of the rest-of-the-world's demand function for the country's exports, for the slope of this function, to be positive. Barring the limiting case when $d=0$, we can regard d to be negative, thus ensuring that the preference function *is* maximized at

1) $ax+by=T$
2) $kx+y\ =S$

Equation *1*) may be called equation of the optimum *expansion-path* for the country. It is the locus of points on what may be called the "iso-preference field" on the (x, y) plane where $dy/dx=k$, *i.e.*, locus of the points of tangency between "iso-preference curves"[9] and lines $kx+y=S$, S taking different alternative values. The second-order condition for maximization of the preference function for any given S being satisfied, the iso-preference curves are strictly convex[10].

The expansion path shows how the country should plan the expansion of its agricultural and industrial output in a one-period plan[11]. Once the size of available investible resources, measured by S, is assessed, the production-possibility function (Equation *2*) is known; intersection between the expansion path and the production-possibility function gives the optimum combination of x and y that is being sought.

IV: ANALYSIS OF THE SOLUTION

The following qualitative analysis of the solution is done with respect to the more pertinent case of $k < P_o$, *i.e.*, the case when the production-opportunity cost of a unit of agricultural good is less than its base-period (export) price, so

[9]Analogous to "indifference curves" in theory of maximization of consumer satisfaction.

[10]The equation of an "iso-preference curve" for a given value $\bar{Z}$ of the preference function is :

$w_{11}x^2+w_{22}y^2+w_{12}xy+w_1x+w_2y-\bar{Z}=0$, which traces out a parabola since $(w_{12}/2)^2=w_{11}\cdot w_{22}$.

[11] In a multiperiod plan also, under certain assumptions. *See*, Section VII.

that the country may be said to have a *comparative advantage* in the production of agricultural good[12].

Nature of Expansion Path

The slope of the expansion path is given by —a/b. Being the marginal export-propensity with respect to a change in agricultural output, 'a' is likely to be positive and is so assumed: a rise in agricultural output, *ceterus paribus*, is expected to increase the supply of exports (of agricultural goods).

The sign of 'b', the marginal export-propensity with respect to a change in industrial output, is negative if the agricultural good is a 'normal' commodity in the country in an aggregative sense. In such a case, a rise in real income resulting from a rise in domestic output, *ceterus paribus*, will be "spent" partly on the industrial good and partly on the agricultural good. This implies that if real income rises through a rise in industrial output, *ceterus paribus*, a contraction of the country's exports (of agricultural good) will result.

If, on the other hand, the agricultural good happens to be an 'inferior' commodity, 'b' will be positive : a rise in real income resulting from a rise in domestic output, *ceterus paribus*, will increase the country's exports (of agricultural good) as a substitution of agricultural good in favour of the industrial good takes place in country's expenditure account.

With 'b' negative (normal-good case), the expression $a—kb > 0$. Even if 'b' were positive (the inferior-good case), it would be reasonable[13] to expect $a—kb > 0$ for the following reason: For any given rise in the expenditure (absorption) on industrial goods consequent to a *given* rise in real income, *less* imports of this good and hence less exports of agricultural good would be required if the given rise in real income takes place through a rise in industrial rather than agricultural output. This means that $a > bP_o$, whence

$$a—kb = a—\frac{k}{P_o} . bP_o > 0, \text{ since } k < P_o.$$

As for the intercepts of the expansion path, they depend, in addition to the signs of 'a' and 'b', on the sign of T. The different possibilities and their implications for optimization are noted below, bearing in mind that

$$T = \frac{(d—c)^2}{2d(a—kb)} . (R—P_o) \gtreqless 0 \quad \text{if} \quad R \gtreqless P_o$$

$$\text{since} \quad \frac{(d—c)^2}{2d(a—kb)} < 0 \quad \text{as} \quad (a—kb) > 0 > d.$$

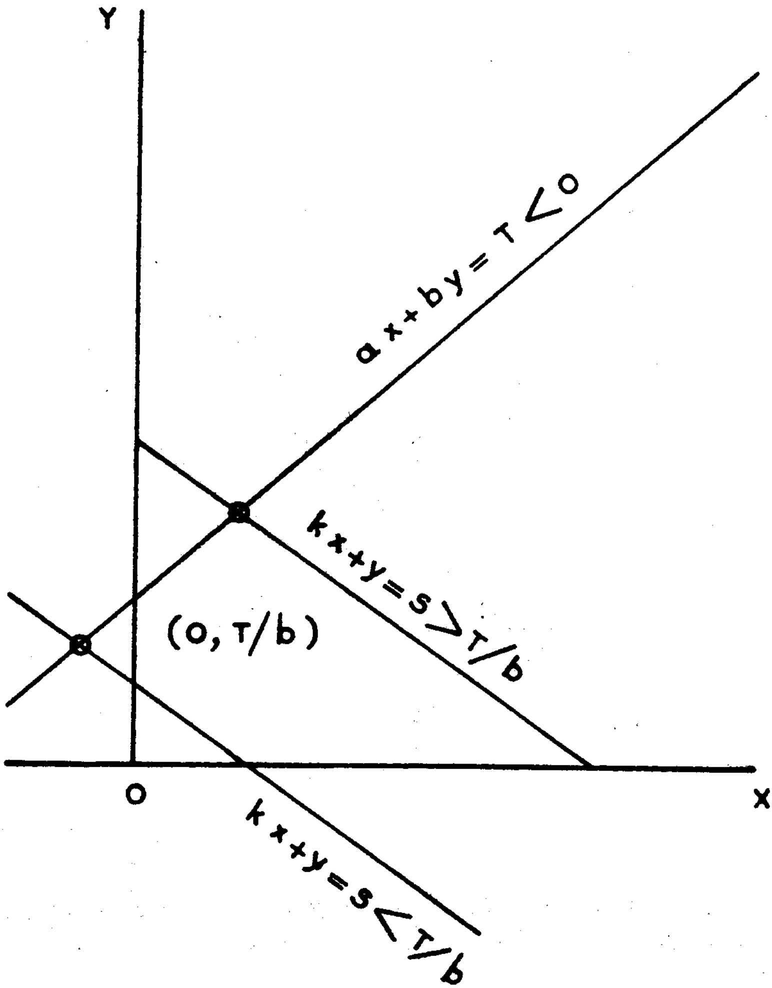
[12] The solution is completely general and is independent of this and other assumptions that follow.

[13] In the theory of consumer equilibrium with two commodities, one inferior good b is necessarily less than 'a' since the position of equilibrium after exchange is independent of whether real income changes through a rise in the initial possession of one or the other commodity.

The Normal-Good Case $(a>0>b)$

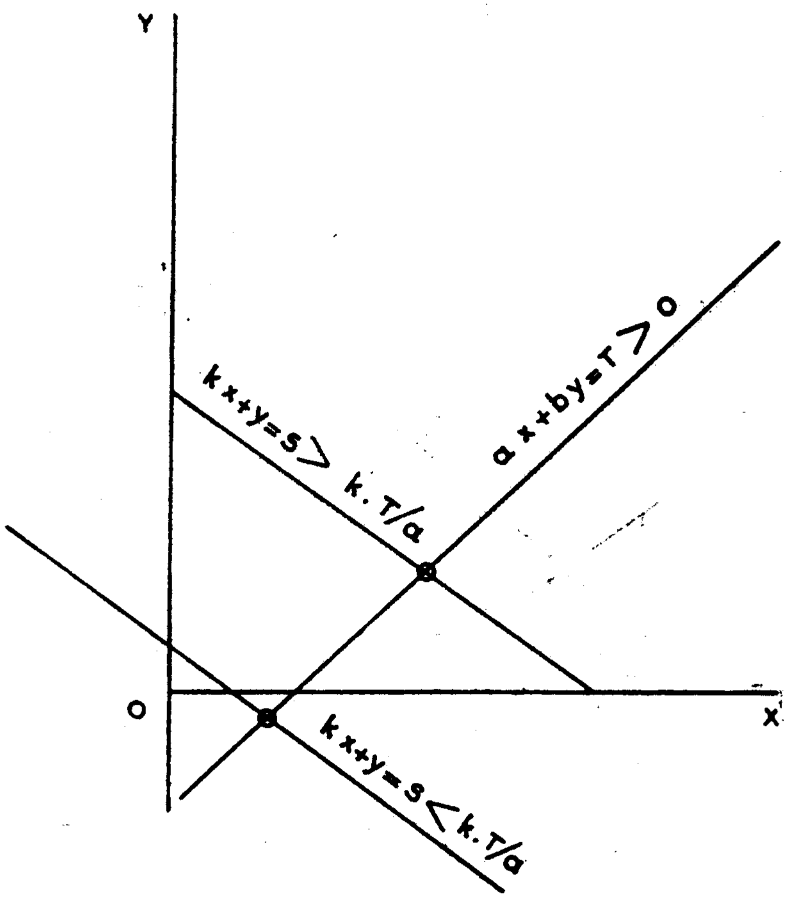
Case 1 $(R>P_o)$: The expansion path in this case slopes upwards to the right with a positive y-intercept (Figure I). Its intersection with the production-possibility function gives $y>0$ for any positive S, whereas $x \gtreqless 0$ for $S \gtreqless T/b$. Thus, optimization requires industrial expansion for any positive investment; agriculture should be expanded simultaneously, if investment is sufficiently large $(S > T/b)$ and should be contracted otherwise $(S < T/b)$.

FIGURE I



Case 2 ($R<P_o$): The expansion path slopes upwards to the right with a positive x-intercept (Figure II). Optimization requires agricultural expansion for any positive investment; industrial expansion, simultaneously, should be planned if investment is sufficiently large ($S>k{\cdot}T/a$), while industrial contraction should be planned if investment is small ($S < k{\cdot}T/a$).

FIGURE II

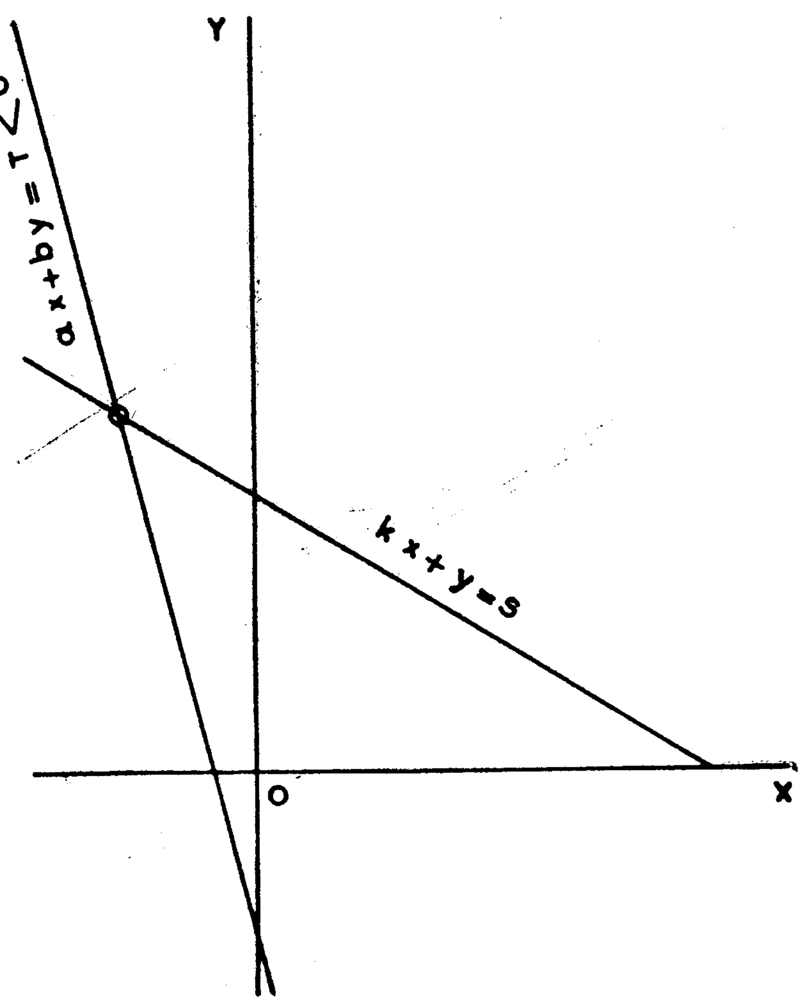


Case 3 ($R=P_o$): The expansion path passes through the origin and both industrial and agricultural expansion should be planned for any positive investment.

The Inferior-Good Case ($a>b>0$)

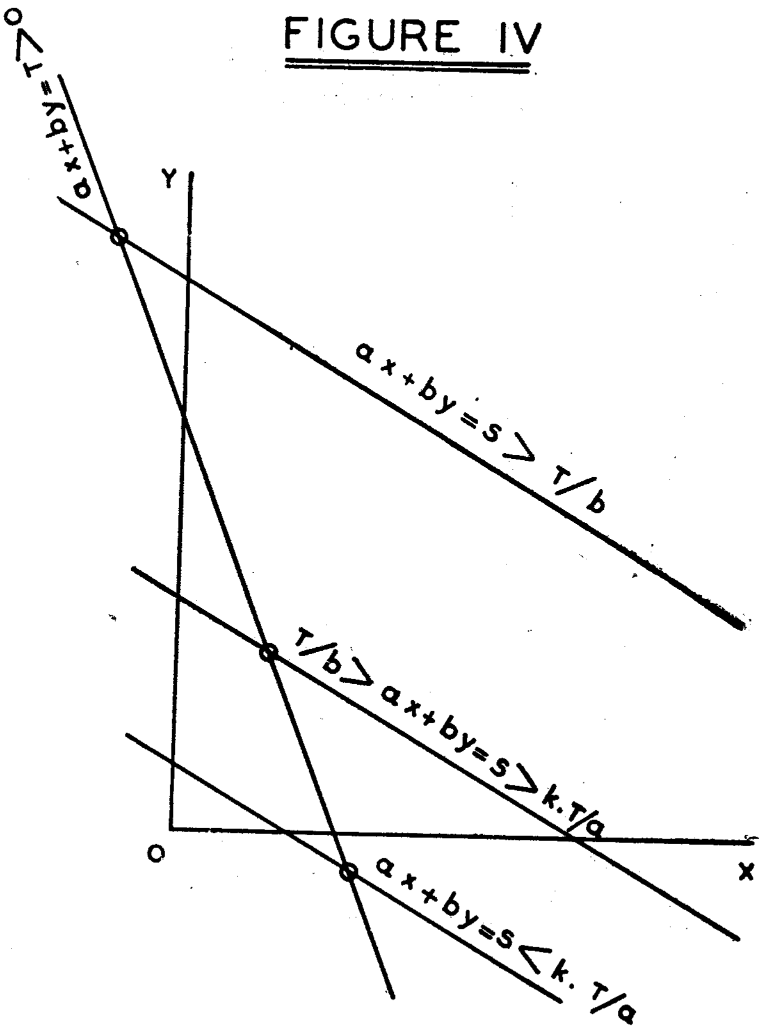
Case 4 ($R \geqslant P_o$): The expansion path slopes downwards to the right with negative or zero intercepts. Moreover, since $a/b > k$, the expansion path, is steeper than the production-possibility function (Figure III), so that for any positive investment optimization takes place in the second quadrant. Hence, industrial expansion and agricultural contraction are to be planned for any positive investment.

FIGURE III



Case 5 ($R < P_o$): The expansion path, sloping downwards to the right and steeper than the production-possibility function, has positive intercepts (Figure IV). For small investment ($S < k \cdot T/a$) agricultural expansion with industrial contraction is required; for larger investment ($T/b > S > k \cdot T/a$) both agricultural and industrial expansion; for still larger investment ($S > T/b$) industrial expansion with agricultural contraction should be planned.

FIGURE IV



The different possibilities are put together in the following summary-table:

Structure	*Industrial planning required*	*Agricultural planning required*	
		Normal-good case	*Inferior-good case*
$R>P_o$	Expansion	Expansion if $S>T/b$ Contraction if $S<T/b$	Contraction
$R=P_o$	Expansion	Expansion	Contraction
$R<P_o$	Expansion if $S>k \cdot T/a$ Contraction if $S<k \cdot T/a$	Expansion	Expansion if $S<T/b$ Contraction if $S>T/b$

It is seen that the type of planning required depends, for the industrial sector, on the size of investment and on whether the expression 'R' exceeds, equals or falls short of, P_o ; for the agricultural sector, it depends, in addition, on whether the agricultural good is a normal or an inferior good in the country.

The expression $R=k+\dfrac{P_o(a-kb)}{e_s-e_d}$ can be identified with what may be called the *marginal real opportunity cost of agricultural good* in terms of industrial good at the pre-plan (initial-period) output-point, *i.e.*, at $x=0=y$, which also lies on the production-possibility function $kx+y=S=0$. This is discussed in the following subsection.

Marginal Real Opportunity Cost of the Agricultural Good

As we move along the production-possibility function $y=S-kx$, the change in real income, z, is given by

$$z=P_o x+y+\frac{P_o(ax+by)}{e_d-e_s}+\frac{d(ax+by)^2}{(d-c)^2} \qquad \text{[\textit{See}, Section IV]}$$

$$=(P_o-k)x+S+\frac{P_o\{(a-kb)x+bS\}}{e_d-e_s}+\frac{d\{(a-kb)x+bS\}^2}{(d-c)^2}$$

[by substituting S—kx for y].

whence

dz/dx (given $y=S-kx$)

$$=P_o-k+\frac{P_o(a-kb)}{e_d-e_s}+\frac{2d(a-kb)[(a-kb)x+bS]}{(d-c)^2}$$

$$=P_o-\left[k+\frac{P_o(a-kb)}{e_s-e_d}-\frac{2d(a-kb)\{(a-kb)x+bS\}}{(d-c)^2}\right]$$

The first term, P_o, is the base-period price of the agricultural good and gives the rise in the value of agricultural output at constant base-period price when agricultural output rises by one unit. If P_o exceeds the expression within the bracket, then the *net* change in real national income resulting from increasing agricultural output by a unit is positive. The expression within the bracket may, thus, be called the *marginal real opportunity cost of agricultural good,* giving the amount of real income that is foregone by moving along the production-possibility function to produce an extra unit of agricultural good. This real opportunity cost is a combination of the *production-opportunity cost, k,* and an expression in terms of the coefficients of the supply and demand functions for exports which may be termed the *loss of real income through a change in the country's terms of trade* resulting from a unit expansion of agricultural output by shifting resources from industry to agriculture[14].

While the production-opportunity cost, k, is constant by assumption, the loss of real income through a change in the terms of trade varies according to the magnitude of S and the point, on the corresponding production-possibility line, from where a shift of resources from industry to agriculture is considered. Accordingly, the marginal real opportunity cost of agricultural good also varies, and equals $k+\frac{P_o\,(a-kb)}{e_s-e_d}=R$ when the shift is considered from the initial output-point $x=0=y$ along the production-possibility function $kx+y=S=0$.

The comparison between R and P_o indicates whether, *at the margin* corresponding to the pre-plan or initial output-point, a move towards industrial or agricultural expansion would increase real national income. An explanation of the different possibilities noted above follows easily.

V: AUTONOMOUS SHIFTS IN EXPORT SUPPLY AND DEMAND FUNCTIONS

The preceding analysis rests on the assumption that the (expected) supply and demand functions for exports determine price and volume of exports not only for the plan period but also for the base period. The base-period price and quantities, however, are *known* magnitudes and although they would presumably have played a part in the estimation of the plan-period supply and demand functions for exports, the latter, when applied to the base period, may not exactly give the former. The discrepancy may be explained by autonomous forces changing over time [15] and/or by purely random forces.

[14] Note that the reduction in industrial output per unit of expansion of agricultural output is k units, and consequently the supply function for exports shifts by the extent (a—kb).

[15] In particular, the rest-of-the-world demand for a country's exports is likely to change with change in the rest-of-the-world income and also with change in supply of the good concerned by other countries. For example, rest-of-the-world demand for Pakistan's jute, the major foreign-exchange earner of the country, rises or falls with booms and recessions in the jute-consuming countries and also with a fall or rise in jute production in other countries like India and Thailand.

For the purpose of the plan problem as postulated, it is immaterial as to why base-period price and quantities of exports may not be given exactly by the estimated plan-period supply and demand functions. Since P_o and E_o are known, all that matters is the difference, if it exists, between E_o, S_{E_o} and D_{E_o}, the last two at the base-period price P_o.

Let S_{E_o} at price $P_o = E_o + \delta_1$ and D_{E_o} at price $P_o = E_o + \delta_2$

Then $S_{E_1} = ax + by + cp + E_o + \delta_1$ [*See*, Section IV]

and $D_{E_1} = dp + E_o + \delta_2$,

so that the balance equation $S_{E_1} = D_{E_1}$ gives $p = \dfrac{ax + by + (\delta_1 - \delta_2)}{(d - c)}$

Working through the analysis of Section IV, it can be shown that the expansion path in this case is given by $ax+by = T'$, with

$$T' = \frac{(d-c)^2}{2d\,(a-kb)}\,[\,R' - P_o\,],\ \text{where}$$

$$R' = k + \frac{P_o\,(a-kb)}{e_s - e_d} + \frac{\delta_2(a-kb)}{c-d} - \frac{2d(\delta_1 - \delta_2)(a-kb)}{(d-c)^2}$$

measuring the marginal real opportunity cost of the agricultural good at $x = 0 = y$ for a move along $kx+y = S = 0$.

It is easy to see that all the qualitative results we had in Section IV (as summarized in the table on Page 215) apply in this case also when we replace T and R by T' and R' respectively.

VI: STEP-WISE LINEARITY IN THE PRODUCTION-POSSIBILITY FUNCTION

Linearity of the production-possibility function irrespective not only of the magnitudes of x and y but also of their *signs* is possibly too unrealistic. It is unlikely, for example, that the production-opportunity cost of agricultural good would be the same whether agricultural output is increased with the help of 'fluid' resources not yet invested in any direction, or by *withdrawing* resources from industry and employing these in agriculture. In the latter case, particularly in the short run, the production-opportunity cost of agricultural good would in general be higher than in the former. And, conversely for withdrawal of resources from agriculture to be used in industry.

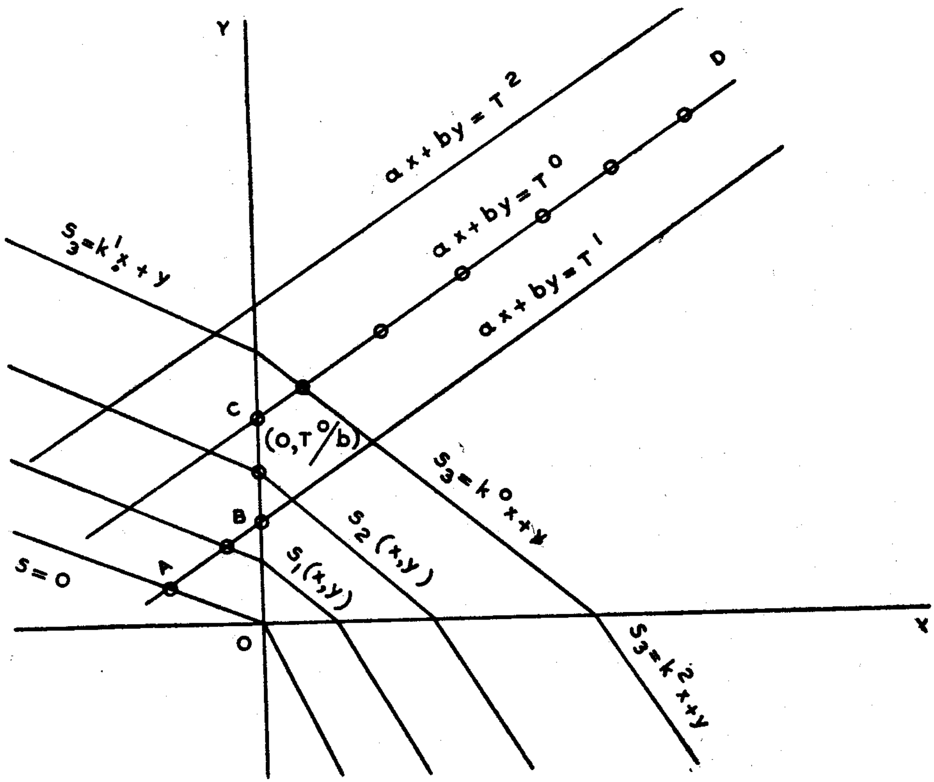
While for the planning task to be manageable, one may want to keep within the bounds of linear techniques, to be reasonably realistic the production-possibility function should be taken to consist of at least three linear segments: the part lying in the quadrant $y > o > x$ would be flatter and that in the quadrant

$x>o>y$ would be steeper than the part in the quadrant $x, y \geqslant o$. The general form of such a step-wise linear function may be written as:

$$S = k^{o}x + y \quad \text{for } x, y \geqslant o;$$
$$S = k^{1}x + y \quad \text{for } y > o > x;$$
$$S = k^{2}x + y \quad \text{for } x > o > y;$$
$$k^{1} > k^{o} > k^{2}$$

Such step-wise linearity, however, does not alter the general moral we have derived from the preceding analysis. This is illustrated graphically (Figure V) in the case where the expansion path $ax + by = T^{o}$ corresponding to $S = k^{o}x + y$ slopes upwards to the right with a positive y-intercept.

FIGURE V



We can picture two other expansion paths, one corresponding to $S = k^{1}x + y$ and the other to $S = k^{2}x + y$. All these three 'basic' expansion paths as they may be called, have the same slope. The intercepts are different, following in general[16] the same order as the k's[17]. The following different situations are possible:

a) If the y-intercept of the production-possibility function falls below the lowest 'basic' expansion path ($ax+by=T^1$), like $S_1(x, y)$ in Figure V, optimization takes place on this expansion path.

b) If the production-possibility function meets the y-axis inbetween the two lower 'basic' expansion paths, like $S_2(x, y)$, optimization takes place on the y-axis.

c) If the production-possibility function meets the y-axis above the middle 'basic' expansion path $ax+by=T^\circ$, then optimization takes place on this expansion path.

The highest expansion path ($ax+by=T^2$) is inoperative in the situation pictured.

This gives the step-wise linear expansion path ABCD for the step-wise linear production-possibility function. This may be compared with the situation where the whole range of the production-possibility function was given by $S=k^\circ x+y$ so that $ax+by=T^\circ$ would give the expansion path for any value of S. The difference starts from point C downwards, and is explained by the extra cost involved in shifting resources already in use from agriculture to industry. The general directive, namely, to concentrate investment in industry as long as $S<T^\circ/b$ and to plan for simultaneous agircultural expansion only when S exceeds T°/b, remains the same in both cases.

VII: A MULTIPERIOD EXTENSION

A multiperiod extension of the analysis easily follows if we abstract from

a) the extra cost of shifting resources already in use from one direction to another as discussed in the previous section;

b) autonomous shifts in the estimated supply and demand functions for exports[18];

and if we make the heuristic assumption, as is in general done in actual national planning in underdeveloped countries, that domestic saving[19] in each

---

16 Except if there is a large autonomous shift in the demand for exports (δ_2) so as to make $\frac{\delta R'}{\delta K}<o$ (*see*, expression for R′ in Section V).

17 The three expansion paths may not all have positive y-intercepts. In the figure they have been so pictured merely for illustration; other possibilities can be handled without any difficulty.

18 It is not necessary to assume that base-period export price and quantities also are given by the estimated supply and demand functions.

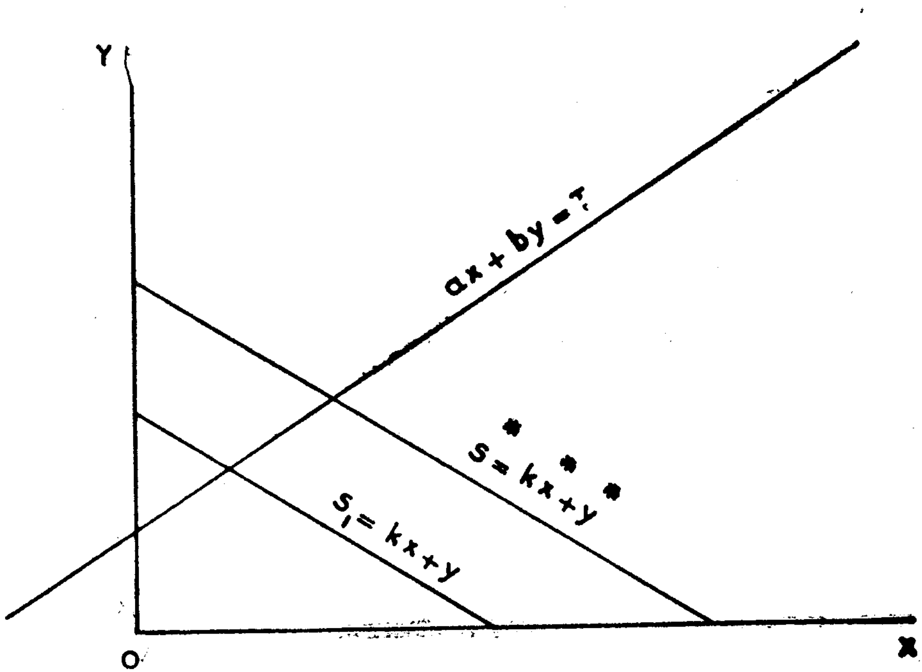
19 Foreign aid, if any, is regarded as exogenously given.

period as a proportion of real national income of that period is independent of the sectoral composition of national output.

Take first, a two-period plan aiming at maximizing real national income $Z_{t=2}$.

Consider, to start with, a *given total* volume of investment for the two periods, denoted by S^* which when utilized increases agricultural output by x^* and industrial output by y^*. Irrespective of how this total investment is distributed between the two periods concerned, the optimum combination of x^* and y^* so as to maximize the corresponding real national income $Z_2 = Z^* = P_o(X_o + x^*) + (Y_o + y^*) + (P^* - P_o) \cdot E^*$ is the same; for optimization must take place on the production-possibility function $S^* = kx^* + y^*$ (Figure VI) and real national income corresponding to any point on this function depends on where this point lies irrespective of what X_1 and Y_1 are.

FIGURE VI



Let S_1 be the share of the first period out of total investment S^*, the share of the second period being $S_2 = S^* - S_1$. Since the distribution $S_1 = S^*$, $S_2 = 0$, which gives the one-period problem, maximizes $Z_2 = Z^* = Z_1$ on the expansion path $ax+by = T$ or $ax+by = T'$ (if the situation discussed in Section V holds) obtained from our analysis of Section III, it follows that Z^* is always maximized on the same expansion path irrespective of how S^* is distributed between S_1 and S_2. The argument applies to any given value of $S^* = S_1 + S_2$.

Hence, for any given total investment over the two periods, real income of Period 2 is maximized on the same expansion path which corresponds to the one-period maximization problem.

Now since S_2, the (estimated) volume of investment available in Period 1 for expanding output in Period 2, is a monotonically rising function of real national income of Period 1, it follows that for any given S_1, S_2 and hence S^* is maximized when real national income Z_1 is maximized. It can be immediately seen that this also maximizes Z_2 when S_2 is utilized optimally.

The analysis easily extends to $t>2$.

Thus, under the assumptions noted above, multiperiod optimum is given by a succession of one-period optima, and the country should plan to move along the same expansion path throughout[20].

It follows that whatever may be the prescription for the initial period or periods, *eventually*, as it grows along the expansion path given by its structure, the country should plan for both industrial and agricultural expansion in the normal-good case; in the inferior (agricultural)-good case, an eventual contraction of agricultural output has to be planned.

If it is intended to recognize a variation of the domestic rate of saving with variation in the sectoral composition of national output, industrial expansion would perhaps be required all the more *earlier* inasmuch as the rate of saving in the industrial sector is presumably higher than that in the agricultural sector.

VIII: CONCLUSION

The results of the study may be contrasted with the classical comparative-cost theory and the subsequent comparative-cost outlook which favours specialization in the line of comparative advantage in each country. Even in the constant-cost case where the comparative-cost outlook has the strongest theoretical support, the present study shows that specialization in primary products, the line in which primary-producing countries presumably have a comparative advantage, is not necessarily in the best interest of these countries initially, and

[20] This is not necessarily so in the step-wise linear production-possibility function case nor if the rate of saving depends on the sectoral composition of national output.

in any case an eventual expansion of the industrial sector is a necessity. *The case for early industrialization is stronger: the greater is planned investment, i.e., the faster is the rate of growth of real national income intended*[21].

The comparative-cost theory, of course, has total *world output,* and not real income of any individual country, as the preference function to be maximized. This, in fact, accounts for its difference with the theory outlined in this study. World output-maximization, however, has little relevance to any individual country in the absence of a satisfactory system of distribution of this output and offers little consolation to underdeveloped countries, particularly to those that are struggling simply for a respectable existence. If such countries strive, as most of them are doing, for maximization of the growth of real national incomes of their own, comparative-cost consideration is only one among a number of factors that should be taken into account. The other factors are the marginal export-propensities of the country with respect to changes in its primary and industrial outputs, and the slopes and price elasticities of the supply and demand functions for its exports, which along with the volume of its primary and industrial outputs would determine the purchasing power of its exports. The inadequacy of the comparative-cost consideration alone for obtaining a directive as to whether a primary-producing country should move towards industrialization is suggested in some recent literature[22]. A rigorous presentation of the case for industrialization in terms of a well-defined and meaningful preference function has, however, so far been awaited.

---

21 Exception to this general rule may be conceived if, for example, a *continuous* positive shift in the rest-of-the-world demand for its primary exports is *anticipated* by planners of the country. But if such shifts were actually to occur without being foreseen at the time of formulating the country's long-term or "perspective" plan, rational planning would still require a move towards industrialization.

22 *See,* for example:

i) R. Prebisch, "The Role of Commercial Policies in Underdeveloped Countries", *American Economic Review* (Papers and Proceedings), May 1950.

ii) H. Singer, "The Distribution of Gains Between Investing and Borrowing Countries", *American Economic Review* (Papers and Proceedings), May 1950.

Note however that insofar as the controversy around the socalled "Singer-Prebisch thesis" is focussed around the *historical* trend of the terms of trade between agriculture and industry. this is not directly relevant to the present study; what concerns us here is whether and to what extent a *planned expansion* of agricultural output should be launched.