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Demand Response in Pakistan: A Modification of the Linear Expenditure System for 1976

EHTISHAM AHMAD, STEPHEN LUDLOW and NICHOLAS STERN*

While demand estimates are used in policy making in a number of areas, there has not been a substantial literature on demand patterns and responses in Pakistan. We present estimates for thirteen classes of goods, based on a complete demand system, a modification of the Linear Expenditure System, using maximum likelihood techniques and observations at the household level for Pakistan and urban and rural areas for 1976.

1. INTRODUCTION

In this paper, we present, estimates of a complete demand system for Pakistan based on cross-section household-level data from the 1976 Micro-Nutrient Survey (MNS) conducted under the aegis of the Planning Commission. These data do not include information on price variation for all commodities, and we therefore impose a functional form which allows us to identify the complete demand system without such information. The resulting price elasticities are, of course, very strongly influenced by the functional form chosen and, as Deaton (1987) puts it, are essentially derived by "assumption". In Section 2 we describe the method used, which is an adaptation of the Extended Linear Expenditure System (ELES) as described in Lluch, Powell and Williams (1977). Our method differs from the standard ELES formulation in that we do not use the income information from the MNS, which we believe to be particularly unreliable [see Ahmad, Leung and Stern (1984)]. We discuss the data further in Section 3, along with the estimated parameters and elasticities. Section 4 concludes.

*The Author is Director, Development Economics Research Programme at the London School of Economics (LSE), currently on leave at the World Bank, Stephen Ludlow is Research Officer at the LSE, and Nicholas Stern is Professor of Economics at the LSE and Chairman, Suntory-Toyota International Centre for Economics and Related Disciplines. They are grateful for grants from the UK Economic and Research Council and the World Bank (RPO 673-13). This paper draws on Ahmad, Leung and Stern (1984) and the acknowledgements in that paper apply here as well.

2. THE LINEAR EXPENDITURE SYSTEM AND MODIFICATIONS

Consider the following standard specification of the Linear Expenditure System (LES):

$$X_i = \gamma_i + b_i(M - \sum_{j=1}^{n} \gamma_j), \quad i = 1, \ldots, n \qquad \ldots \qquad \ldots \qquad (1)$$

where

$$\sum_{i=1}^{n} b_i = 1. \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (1a)$$

where X_i is expenditure on the ith good, and M is total money expenditure, with $\Sigma_i X_i = M$ (for $i = 1, \ldots, n$). The γ_i, when positive, could be taken to represent the value of the "minimum consumption requirements" of the ith commodity, and $(M - \Sigma_j \gamma_j)$ represents the "supernumerary consumption". Note that negative γ_i are possible and do not yield a simple intuitive explanation, beyond suggesting that such commodities are unlikely to represent "essential consumption". However $\Sigma_j \gamma_j$ represents the total subsistence consumption requirement. The b_i's are often referred to as the marginal budget share.

As it stands the structure of the demand system Equation (1) is underidentified. We may see this by posing the question of identification of a simultaneous system, as the derivation of the structure from the reduced form:

$$X_i = a_i + b_i M \quad (i = 1, 2 \ldots, n) \qquad \ldots \qquad \ldots \qquad \ldots \qquad (2)$$

The reduced form Equation (2), ignoring the random term, would provide us with $(2n-2)$ independent parameters amongst the a_i and b_i since the adding-up constraint $\Sigma_i X_i = M$ $(i = 1, \ldots, n)$, which holds for each household, will imply

$$\sum_{i=1}^{n} a_i = 0, \quad \text{and} \quad \sum_{i=1}^{n} b_i = 1 \qquad \ldots \qquad \ldots \qquad \ldots \qquad (2a)$$

Thus the standard procedure in estimating a system such as Equation (2) is simply to drop one equation. From this set of $(2n - 2)$ parameters in the reduced form we wish to construct $(2n - 1)$ parameters in the structure – there are n of the γ_i and $(n - 1)$ independent b_i. The b_i are identified but the γ_i are not. Essentially we require one additional restriction to identify.

The standard way of achieving identification in this context is to set one of the γ_i to be zero. In particular in the ELES (where the extension is the inclusion of an equation for saving so that M is total income rather than total expenditure) it is the γ_i for saving which is set at zero although the role of this assumption is not always explicit [see e.g. Ali (1985)].

We do not, however, have confidence [see Ahmad, Leung and Stern (1984)] in the income variable in the MNS – this is nototiously hard to measure in household surveys in developing countries – and we do not therefore follow the ELES procedure. Instead we introduce the information on the number of members or household size in the MNS to provide extra parameter estimates. specifically we suppose that the "minimum consumption requirments" depend on household size n. This is expressed as:

$$\gamma_i = nd_i + f_i \quad (i = 1, 2, \ldots, n) \qquad \ldots \qquad \ldots \qquad \ldots \qquad (3)$$

where d_i and f_i are parameters relating "minimum consumption requirements" to household size. For the consumption of commodities independent of household size, $d_i = 0$, and $f_i = 0$ for the consumption of commodities proportional to household size. If we now examine the reduced form:

$$X_i = a_i + b_i M + g_i n \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (4)$$

we can see that we have $3n - 3$ independent parameters amongst the a_i, b_i and g_i (note that adding-up will imply $\Sigma_i a_i = \Sigma_i g_i = 0$, and $\Sigma_i b_i = 1$, for $i = 1, 2, \ldots, n$). From these we want to calculate the $(3n - 1)$ parameters (the b_i, d_i, f_i) in the structure formed by Equations (1) and (3). Thus we are now 2 identifying restrictions short. However, we can achieve identification by imposing restrictions on the parameters of Equation (3). Specifically we think of some of the γ_i as being proportional to household size so that f_i is zero, others as being independent of household size so that d_i is zero. Thinking of a household as a small community, f_i zero means i is analagous to a private good for each member (we must all eat), and d_i equal to zero means i is analagous to a public good (an individual's use of the broom or refrigerator does not diminish its availability to other members of the household).

The list of 13 commodities which we use for estimation is contained in Table 1 and we suppose that the minimum requirements for all commodities except (10) 'housing', and (13) 'other non-food' are proportional to household size ($f_i = 0$ for $i \neq 10, 13$), and minimum requirements for goods (10) and (13) are independent of household size ($d_i = 0$ for $i = 10, 13$). Thus we estimate the system (5) where ϵ_i is a random term.

$$X_i^h = n^h \cdot d_i + b_i (M^h - \Sigma_j n^h \cdot d_j - \gamma_{10} - \gamma_{13}) + \epsilon_i,$$

for $i, j = 1, \ldots, 9, 11, 12$

$$X_i^h = \gamma_i + b_i (M^h - \Sigma_j n^h \cdot d_j - \gamma_{10} - \gamma_{13}) + \epsilon_i,$$

for $i = 10, 13; j = 1, \ldots, 9, 11, 12$

with

$$\sum_{i=1}^{n} b_i = 1. \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (5)$$

Table 1

Parameter Estimates for Modified LES

		Urban and Rural 975 Cases		Zero Observations	Urban 459 Cases		Rural 516 Cases	
		$b(i)$	d_i and γ_i		$b(i)$	d_i and γ_i	$b(i)$	d_i and γ_i
1.	Wheat	0.027	10.873	16	0.025	8.159	0.054	10.888
		(0.003)	(0.349)		(0.003)	(0.373)	(0.006)	(0.685)
2.	Rice	0.031	2.092	125	0.026	2.050	0.050	0.4135
		(0.002)	(0.301)		(0.003)	(0.398)	(0.005)	(0.562)
3.	Pulses	0.007	1.739	26	0.006	1.772	0.012	1.235
		(0.001)	(0.101)		(0.001)	(0.141)	(0.002)	(0.185)
4.	Meat and Eggs	0.121	–1.142	83	0.132	–1.326	0.080	–1.092
		(0.003)	(0.616)		(0.004)	(1.059)	(0.006)	(0.719)
5.	Milk	0.127	6.297	69	0.108	4.666	0.197	1.229
		(0.006)	(0.957)		(0.007)	(1.221)	(0.012)	(1.863)
6.	Vegetables, Fruits and Spices	0.118	1.176	2	0.120	2.270	0.101	–1.014
		(0.003)	(0.616)		(0.004)	(1.024)	(0.005)	(0.762)
7.	Edible Oils	0.076	3.378	33	0.074	2.736	0.083	1.964
		(0 003)	(0.514)		(0.004)	(0.081)	(0.006)	(0.794)
8.	Sugar	0.025	2.517	85	0.020	3.815	0.032	0.589
		(0.001)	(0.184)		(0.002)	(0.251)	(0.002)	(0.310)
9.	Tea	0.013	0.977	24	0.014	0.822	0.009	1.031
		(0.001)	(0.095)		(0.001)	(0.150)	(0.001)	(0.143)
10.	Housing	0.067	3.361	105	0.077	11.095	0.028	–0.676
		(0.004)	(3.169)		(0.005)	(5.581)	(0.005)	(3.240)
11.	Clothing	0.114	4.788	5	0.112	4.244	0.120	2.469
		(0.004)	(0.663)		(0.005)	(1.032)	(0.007)	(0.988)
12.	Other Food	0.33	2.359	144	0.033	0.735	0.047	2.125
		(0.003)	(0.324)		(0.003)	(0.496)	(0.004)	(0.493)
13.	Other Non-food	0.241	–16.466	21	0.254	–16.448	0.187	–27.118
		(0.006)	(7.236)		(0.008)	(12.665)	(0.008)	(7.007)
	2*Log-Likelihood	–8723.4			–4202.9		–3149.6	

Continued –

Table 1 – (*Continued*)

(Standard errors in parentheses)

Notes: (i) The $b(i)$ are the marginal budget shares.

(ii) The entries in the columns headed d_i and γ_i are d_i for $i \neq 10, 13$ and γ_i (equal to f_i) for $i = 10, 13$ – see text, particularly Equation (3) and the subsequent discussion. The units for d_i are rupees per head and for γ_i are rupees per month.

(iii) "Other food" consists of Maize and Other Cereals, Sweets, Beverages, Gur and other food.

Having imposed n restrictions on the d_i and f_i we have $n - 2$ over-identifying restrictions. Accordingly there are $(n - 2)$ restrictions on the reduced form.

The structure (5) was estimated directly using Deaton's NLFIML programme for a maximum likelihood estimation of a complete demand system [see Deaton (1981)]. The residuals are assumed to be drawn from a multinormal distribution with mean vector 0 and variance V. The contemporaneous variances and covariances make up the constant matrix V. We have not given any special treatment to zero purchases and have simply included all observations for the relevant sample. Zero purchases are generally below 15 percent of the sample of 975 cases, and are as low as 2 households for the commodity (6) 'vegetables, fruit and spices' (see Table 1). The estimates are presented in the next section.

This "modified" LES may be improved upon in a number of ways. For instance, one can bring in other household characteristics such as the number of adults or children (Barten 1964). Then the "minimum requirements" on tobacco or other adult goods could be made to depend on the number of adults, whilst those on education or other "children's goods would be dependent on the number of children. Similarly, one might distinguish between predominantly male or female commodities. Unfortunately the 1976 MNS does not allow further refinements in the manner described.

The standard method of identifying a modified LES in the absence of price information [see Equations (2) and (3) above, and also Ahmad, Leung and Stern (1984) for experiments with a 17-commodity classification for Pakistan] would require, for instance, that the minimum consumption requirements for a particular commodity group be specified *ex-ante*. The introduce an element of sensitivity we have reestimated the modified LES with the identifying assumption that the minimum requirement for the commodity group 'other non-food' is zero (see Appendix Table 1). We do not report the full set of results here and comment only on the differences that arise in identifying the LES differently in the sections below.

3. DATA AND ESTIMATES

The 13 commodity groups derived from the 1976 Pakistan MNS data tapes are: (1) 'wheat'; (2) 'rice'; (3) 'pulses'; (4) 'meat, fish and eggs'; (5) 'milk and products'; (6) 'vegetables, fruit and spices'; (7) 'edible oils'; (8) 'sugar', (9) 'tea'; (10) 'housing' (including durables like furnishing and utensils; and fuel and light, such as gas, electricity and water); (11) 'clothing' (including shoes, laundry and repairs); (12) 'other food' (including maize and other cereals, sweets, gur, and other food); and (13) 'other non-food' (including cosmetics, tobacco, education, recreation, personal hygiene and so on). This classification is determined by the MNS data set. It is not possible to disentangle, say 'fuel and light' from (10) 'housing', or 'tobacco' from (13) 'other non-food', although the demand response with respect to such commodities may have considerable importance in estimates of tax revenues or for other purposes. A finer commodity classification is possible, however, with the 1979 Household Income and Expenditure Survey, and this will be the subject of further work.

We present estimates for parameters based on the full 975-household sample, along with estimates for urban households and rural households separately in Table 1. It is immediately apparent from the marginal budget shares, b_i, that there are significant differences between the urban and rural demand patterns. For instance, the marginal budget shares for food-grains [commodities (1)–(3)] in rural areas are considerably higher than for urban households. Correspondingly, urban households have higher marginal budget shares in the case of (4) 'meat, fish and eggs'; (6) 'vegetables, fruits and spices'; (10) 'housing'; and (13) 'other non-food'. There are thus limitations that need to be kept in mind in using the aggregate parameters based on the full sample, given the differences in the patterns of demand. Within a given sector there may be further differences on the basis of income groups [see Radhakrishna and Murty (1980)] and these will be examined further with the 1979 Household Income and Expenditure Survey (HIES) for Pakistan which has a much larger sample and finer commodity classification than the MNS. The differences in demand patterns across rural and urban areas may, in part, reflect such income differences. However these may also be due to different taste patterns, regional differences, availability o' items (such as gur) and so on. An investigation of this would require piece-wise linear estimates for different income groups within rural and urban areas separately and will be attempted with the larger sample-size of the HIES data for later years.

The uncompensated own-price elasticity η_{ii} is given by:

$$\eta_{ii} = (1-b_i)\frac{\gamma_i}{X_i} - 1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (6)$$

where $0 < \gamma_i < X_i$ and the elasticity is negative with an absolute value between b_i and 1. The uncompensated cross-price elasticity η_{ij} is given by

$$\eta_{ij} = -\frac{b_i\gamma_j}{X_i} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (7)$$

where X_i is the expenditure on the ith good. When $\gamma_i > 0$ then $\eta_{ij} < 0$, i.e. all goods are gross complements. We present the uncompensated own-price elasticities for the full, urban and rural samples evaluated at sample means in Tables 2 – 4. The own-price elasticities are all negative. They are also predominantly less than unity in absolute value, with the exception of (4) 'meat, fish and eggs', and (13)'other non-food', and these correspond to the negative values of the γ_i's as seen in Table 1. Note

Table 2

Uncompensated Price Elasticities (full sample)

Row \ Col.	Wheat (1)	Rice (2)	Pulses (3)	Meat and Eggs (4)	Milk (5)	Vegetables, Fruits and Spices (6)	Edible Oils (7)	Sugar (8)	Tea (9)	Housing (10)	Clothing (11)	Other Food (12)	Other Non-food (13)
1. Wheat	–0.23625	–0.00401	–0.00333	0.00219	–0.01207	–0.00225	–0.00648	–0.00483	–0.00187	–0.00106	–0.00918	–0.00452	0.00517
2. Rice	–0.06767	–0.59965	–0.01082	0.00710	–0.03919	–0.00732	–0.02103	–0.01567	–0.00608	–0.00343	–0.02980	–0.01468	0.01680
3. Pulses	–0.03184	–0.00613	–0.29930	0.00334	–0.01844	–0.00344	–0.00989	–0.00737	–0.00286	–0.00161	–0.01402	–0.00691	0.00790
4. Meat and Eggs	–0.15046	–0.02895	–0.02406	–1.11524	–0.08714	–0.01627	–0.04675	–0.03483	–0.01352	–0.00762	–0.06625	–0.03264	0.03735
5. Milk	–0.07817	–0.01564	–0.01250	0.00821	–0.69016	–0.00845	–0.02429	–0.01810	–0.00703	–0.00396	–0.03442	–0.01696	0.01940
6. Vegetables, Fruits and Spices	–0.11387	–0.02191	–0.01821	0.01196	–0.06595	–0.90836	–0.03538	–0.02636	–0.01024	–0.00577	–0.05014	–0.02470	0.02826
7. Edible Oils	–0.08126	–0.01564	–0.01299	0.00853	–0.04706	–0.00879	–0.69304	–0.01881	–0.00730	–0.00412	–0.03578	–0.01763	0.02017
8. Sugar	–0.05852	–0.01126	–0.00936	0.00614	–0.03389	–0.00633	–0.01818	–0.47563	–0.00526	–0.00297	–0.02577	–0.01269	0.01452
9. Tea	–0.06460	–0.01243	–0.01033	0.00678	–0.03741	–0.00698	–0.02007	–0.01496	–0.54747	–0.00327	–0.02845	–0.01402	0.01604
10. Housing	–0.11673	–0.02246	–0.01866	0.01226	–0.06760	–0.01262	–0.03627	–0.02702	–0.01049	–0.91736	–0.05140	–0.02532	0.02897
11. Clothing	–0.08504	–0.01636	–0.01360	0.00893	–0.04925	–0.00919	–0.02642	–0.01969	–0.00764	–0.00431	–0.70787	–0.01845	0.02111
12. Other Food	–0.06593	–0.01269	–0.01054	0.00692	–0.03818	–0.00713	–0.02049	–0.01526	–0.00593	–0.00334	–0.02903	–0.58053	0.01637
13. Other Non-food	–0.14752	–0.02838	–0.02359	0.01549	–0.08543	–0.01595	–0.04584	–0.03415	–0.01326	–0.00747	–0.06496	–0.03200	–1.11532

Table 3

Uncompensated Price Elasticities (urban sample)

Row \ Col.	Wheat (1)	Rice (2)	Pulses (3)	Meat and Eggs (4)	Milk (5)	Vegetables, Fruits and Spices (6)	Edible Oils (7)	Sugar (8)	Tea (9)	Housing (10)	Clothing (11)	Other Food (12)	Other Non-food (13)
1. Wheat	–0.27716	–0.00458	–0.00396	0.00296	–0.01042	–0.00507	–0.00611	–0.00852	–0.00184	–0.00393	–0.00948	–0.00164	0.00582
2. Rice	–0.04284	–0.59681	–0.00931	0.00696	–0.02450	–0.01192	–0.01437	–0.02003	–0.00432	–0.00923	–0.02228	–00.386	0.01369
3. Pulses	–0.01898	–0.00477	–0.27545	0.00308	–0.01085	–0.00528	–0.00636	–0.00887	–0.00191	–0.00409	–0.00987	–0.00171	0.00606
4. Meat and Eggs	–0.09028	–0.02268	–0.01961	–1.09662	–0.05163	–0.02512	–0.03027	–0.04222	–0.00909	–0.01946	–0.04696	–0.00813	0.02885
5. Milk	–0.05598	–0.01407	–0.01216	0.00909	–0.73530	–0.01558	–0.01877	–0.02618	–0.00564	–0.01207	–0.02912	–0.00504	0.01789
6. Vegetables, Fruits and Spices	–0.06691	–0.01681	–0.01453	0.01087	–0.03827	–0.86305	–0.02244	–0.03129	–0.00674	–0.01442	–0.03480	–0.00603	0.02138
7. Edible Oils	–0.05992	–0.01505	–0.01302	0.00973	–0.03427	–0.01667	–0.75039	–0.02802	–0.00604	–0.01291	–0.03116	–0.00540	0.01914
8. Sugar	–0.02702	–0.00679	–0.00587	0.00439	–0.01545	–0.00752	–0.00906	–0.37790	–0.00272	–0.00582	–0.01405	–0.00243	0.00863
9. Tea	–0.04902	–0.01232	–0.01065	0.00796	–0.02803	–0.01364	–0.01644	–0.02292	–0.65118	–0.01056	–0.02549	–0.00442	0.01566
10. Housing	–0.06233	–0.01566	–0.01354	0.01013	–0.03565	–0.01734	–0.02090	–0.02915	–0.00628	–0.84008	–0.03242	–0.00561	0.01992
11. Clothing	–0.05744	–0.01443	–0.01248	0.00933	–0.03285	–0.01598	–0.01926	–0.02686	–0.00579	–0.01238	–0.76368	–0.00517	0.01835
12. Other Food	–0.06164	–0.01549	–0.01339	0.01001	–0.03525	–0.01715	–0.02067	–0.02882	–0.00621	–0.01329	–0.03206	–0.83504	0.01969
13. Other Non-food	–0.08729	–0.02193	–0.01896	0.01418	–0.04992	–0.02428	–0.02927	–0.04082	–0.00879	–0.01881	–0.04540	–0.00786	–1.08201

Table 4

Uncompensated Price Elasticities (rural sample)

Row \ Col.	Wheat (1)	Rice (2)	Pulses (3)	Meat and Eggs (4)	Milk (5)	Vegetables, Fruits and Spices (6)	Edible Oils (7)	Sugar (8)	Tea (9)	Housing (10)	Clothing (11)	Other Food (12)	Other Non-food (13)
1. Wheat	–0.37791	–0.00134	–0.00401	0.00355	–0.00400	0.00330	–0.00639	–0.00190	–0.00335	0.00037	–0.00803	–0.00691	0.01490
2. Rice	–0.10437	–0.92390	–0.01184	0.01046	–0.01178	0.00972	–0.01883	–0.00562	–0.00989	0.00109	–0.02367	–0.02037	0.04394
3. Pulses	–0.05407	–0.00205	–0.51070	0.00542	–0.00611	0.00504	–0.00975	–0.00291	–0.00512	0.00057	–0.01226	–0.01056	0.02276
4. Meat and Eggs	–0.15350	–0.00583	–0.01741	–1.17683	–0.01733	0.01430	–0.02769	–0.00826	–0.01454	0.00161	–0.03481	–0.02996	0.06462
5. Milk	–0.10936	–0.00415	–0.01240	0.01096	–0.94973	0.01019	–0.01973	–0.00588	–0.01036	0.00115	–0.02480	–0.02135	0.04604
6. Vegetables, Fruits and Spices	–0.13390	–0.00509	–0.01519	0.01342	–0.01512	–1.11129	–0.02415	–0.00720	–0.01268	0.00140	–0.03037	–0.02614	0.05637
7. Edible Oils	–0.08882	–0.00337	–0.01007	0.00890	–0.01003	0.00827	–0.82336	–0.00478	–0.00841	0.00093	–0.02014	–0.01734	0.03739
8. Sugar	–0.10143	–0.00385	–0.01150	0.01017	–0.01145	0.00945	–0.01829	–0.83402	–0.00961	0.00106	–0.02300	–0.01980	0.04270
9. Tea	–0.05177	–0.00197	–0.00587	0.00519	–0.00585	0.00482	–0.00934	–0.00279	–0.47632	0.00054	–0.01174	–0.01011	0.02179
10. Housing	–0.12389	–0.00471	–0.01405	0.01242	–0.01399	0.01154	–0.02235	–0.00667	–0.01174	–1.04438	–0.02810	–0.02418	0.05216
11. Clothing	–0.09887	–0.00376	–0.01121	0.00991	–0.01116	0.00921	–0.01783	–0.00532	–0.00937	0.00104	–0.83516	–0.01930	0.04162
12. Other Food	–0.07809	–0.00297	–0.00886	0.00783	–0.00882	0.00727	–0.01409	–0.00420	–0.00740	0.00082	–0.01771	–0.68866	0.03287
13. Other Non-food	–0.16820	–0.00639	–0.01908	0.01686	–0.01899	0.01567	–0.03034	–0.00905	–0.01593	0.00176	–0.03814	–0.03283	–1.30764

that the negative γ_i's are generally insignificantly different from zero. The cross-price effects for most commodities are negative, again the exception being those cases for which the γ_i's are negative and the substitution effects prevail. Apart from the cross-price effects with respect to the price of (1) 'wheat', (5) 'milk' and to some extent (11) 'clothing', the absolute values of the cross-price terms are small.

Given the significantly different patterns of consumption in rural and urban areas it is interesting to evaluate an aggregate demand response using the piece-wise LES (PLES) parameters for the urban-rural subdivision of the sample. The 'composite' aggregate elasticity matrix, with η_{ii} and η_{ij} as the own– and cross-price elements, may be expressed as the weighted averages of the own– and cross-price elasticities of the relevant sub-groups:

$$\eta_{ii} = \sum_c s_i^c \, \eta_{ii}^c \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

$$\eta_{ij} = \sum_c s_i^c \, \eta_{ij}^c \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

and

$$s_i^c = \frac{X_i^c}{\sum_c X_i^c} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

where c denotes the cth class and s_i^c is the proportion of the aggregate consumption of the ith good accounted for by the cth class. The aggregate elasticity matrix based on the PLES estimates for rural and urban sectors is presented in Table 5. This may be compared with the full-sample estimates from Table 2. For instance the own-price elasticity for (1) 'wheat' is –0.34 for the PLES (a weighted average of –0.38 for the rural sector and –0.28 for the urban), whereas the full sample estimate was –0.24.

The expenditure elasticities for rural, urban and all household groups are presented in Table 6. These also reflect differences in the consumption patterns across rural and urban households. These differences are most pronounced in the food grains commodity groups, and for sugar.

Identification of a modified LES on the assumption that the γ_i for (13) 'other non-food' is zero leads to results that are not too dissimilar to those reported above, especially with respect to the marginal budget shares b_i (see Appendix Table 1 for the parameters corresponding to the classifications of the sample as in Table 1). However, since γ_{13} is arbitrarily zero in this formulation, differences arise with respect to the magnitudes of some of the γ_i. Since the elasticities calculated involve both the b_i and the γ_i, there are bound to be differences between the two formulations. However, our experiments suggest that the magnitudes of the own-price effects are not significantly altered, and for some purposes the elasticities may be treated as rough approximations of each other.

Table 5

Uncompensated Aggregate Price Elasticities (piecewise urban and rural)

Row \ Col.	Wheat (1)	Rice (2)	Pulses (3)	Meat and Eggs (4)	Milk (5)	Vegetables, Fruits and Spices (6)	Edible Oils (7)	Sugar (8)	Tea (9)	Housing (10)	Clothing (11)	Other Food (12)	Other Non-food (13)
1. Wheat	−0.33894	−0.00259	−0.00399	0.00332	−0.00648	0.00006	−0.00628	−0.00446	−0.00277	−0.00129	−0.00859	−0.00487	0.01139
2. Rice	−0.07506	−0.76811	−0.01063	0.00879	−0.01784	−0.00059	−0.01671	−0.01248	−0.00724	−0.00383	−0.02301	−0.01251	0.02953
3. Pulses	−0.03720	−0.00336	−0.39762	0.00430	−0.00839	0.00008	−0.00812	−0.00577	−0.00358	−0.00167	−0.01111	−0.00631	0.01473
4. Meat and Eggs	−0.11143	−0.01704	−0.01887	−1.12345	−0.04016	−0.01193	−0.02941	−0.03086	−0.01091	−0.01241	−0.04290	−0.01543	0.04082
5. Milk	−0.08631	−0.00843	−0.01230	0.01015	−0.85714	−0.00094	−0.01932	−0.01465	−0.00832	−0.00456	−0.02667	−0.01431	0.03388
6. Vegetables, Fruits and Spices	−0.09182	−0.01245	−0.01478	0.01182	−0.02966	−0.95537	−0.02308	−0.02233	−0.00895	−0.00854	−0.03315	−0.01351	0.03439
7. Edible Oils	−0.07479	−0.00904	−0.01150	0.00930	−0.02180	−0.00384	−0.78793	−0.01607	−0.00726	−0.00579	−0.02549	−0.01154	0.02853
8. Sugar	−0.05490	−0.00569	−0.00798	0.00656	−0.01395	−0.00116	−0.01252	−0.54881	−0.00530	−0.00324	−0.01740	−0.00894	0.02140
9. Tea	−0.05031	−0.00746	−0.00841	0.00666	−0.01761	−0.00497	−0.01311	−0.01347	−0.56907	−0.00535	−0.01903	−0.00709	0.01854
10. Housing	−0.07502	−0.01340	−0.01365	0.01060	−0.03118	−0.01138	−0.02120	−0.02451	−0.00741	−0.88221	−0.03153	−0.00944	0.02657
11. Clothing	−0.07674	−0.00946	−0.01189	0.00960	−0.02275	−0.00425	−0.01859	−0.01683	−0.00746	−0.00613	−0.79698	−0.01175	0.02919
12. Other Food	−0.07174	−0.00780	−0.01061	0.00867	−0.01902	−0.00215	−0.01663	−0.01370	−0.00694	−0.00463	−0.02325	−0.74515	0.02778
13. Other Non-food	−0.11560	−0.01649	−0.01900	0.01512	−0.03910	−0.01030	−0.02964	−0.02970	−0.01129	−0.01161	−0.04286	−0.01660	−1.16096

Table 6

Expenditure Elasticities

	Total Expenditure Elasticities		
	Urban and Rural	Urban	Rural
1. Wheat	0.2302	0.2975	0.3498
2. Rice	0.7472	0.6995	1.0313
3. Pulses	0.3515	0.3099	0.5343
4. Meat and Eggs	1.6614	1.4741	1.5168
5. Milk	0.8632	0.9141	1.0806
6. Vegetables Fruit and Spices	1.2574	1.0924	1.3231
7. Edible Oils	0.8973	0.9783	0.8777
8. Sugar	0.6462	0.4411	1.0022
9. Tea	0.7134	0.8003	0.5116
10. Housing	1.2890	1.0177	1.2242
11. Clothing	0.9390	0.9378	0.9770
12. Other Foods*	0.7280	1.0064	0.7717
13. Other Non-foods**	1.6289	1.4252	1.6620

Notes. *Maize and other Cereals, Sweets, Beverages, Other Food, Gur.
**Other Non-food, Recreation, Education, Hygiene, Medicine.

4. CONCLUDING REMARKS

There has not been a great deal of work on the analysis of demand in Pakistan [see Ali (1985), for a recent review of the literature], and the most sophisticated analysis published has been the ELES reported by Ali (1985) who fitted single equation OLS estimates for each of 12 commodity groups with grouped cross section data from the 1979 HIES. The parameters in this paper are derived from Maximum Likelihood estimates and along with the earlier work by Ahmad, Leung and Stern (1984) are the first to be based on household observations for Pakistan. We expect to follow this with detailed estimates based on a finer commodity classification from the 1979 HIES data.

The LES has been criticised in the literature [see e.g., Deaton (1987)] on the grounds *inter alia* that the linearity of the Engel curves and the assumption of separability restrict the usefulness of the estimates. The piece-wise linear estimates relax to some extent the first set of criticisms, although within-group linearity is not eliminated. In the Indian context, Ray (1980, 1982) has argued for non-homothetic, non-separable commodity demand functions with non-linear Engel curves. However without price variation it will not in general be possible to estimate non-separable forms. The use of (only) cross-section data ties us to this assumption, unless unit value information is available and in a manner which permits one to distinguish quality variation from genuine price variation [see Deaton (1986)]. On the other hand cross-section data do permit detailed analysis of the impact of price changes and of the way in which demand varies across different types of household. For this the 1979 HIES will be very useful. However no single cross-section is reliable taken in isolation, and the estimates based on the MNS 1976 data here will provide a very useful standard of comparison for future work on the more detailed data set.

There are a number of policy issues which require an estimation of demand responses, including *inter alia* the determination of price levels for various groups of commodities. Our main purpose [see Ahmad and Stern (1986)] is to provide estimates that will be used in the analysis of tax reform, which involves an assessment of the magnitude of consumer response to the tax (or price) changes.

Appendix Table 1

Parameter Estimates for Modified LES

	Full Sample 975 Cases			Urban 459 Cases			Rural 576 Cases		
	b_i	γ_i	R^2	b_i	γ_i	R^2	b_i	γ_i	R^2
1. Wheat	0.262	11.119	0.28	0.025	8.395	0.42	0.053	11.840	0.37
	(0.003)	(0.319)		(0.003)	(0.332)		(0.006)	(0.579)	
2. Rice	0.032	2.330	0.16	0.026	2.33	0.18	0.050	1.245	0.18
	(0.002)	(0.269)		(0.003)	(0.366)		(0.005)	(0.467)	
3. Pulses	0.007	1.788	0.15	0.005	1.85	0.16	0.012	1.452	0.15
	(0.001)	(0.094)		(0.001)	(0.134)		(0.002)	(0.157)	
4. Wheat and Eggs	0.121	–0.224	0.59	0.132	–0.181	0.67	0.081	0.166	0.35
	(0.003)	(0.435)		(0.004)	(0.726)		(0.006)	(0.568)	
5. Milk	0.127	7.372	0.33	0.108	5.536	0.41	0.198	4.536	0.37
	(0.006)	(8.801)		(0.007)	(0.947)		(0.012)	(1.508)	
6. Vegetables, etc.	0.119	2.074	0.61	0.120	3.326	0.64	0.101	0.604	0.48
	(0.003)	(0.447)		(0.004)	(0.745)		(0.005)	(0.593)	
7. Edible Oils	0.076	3.977	0.41	0.075	3.392	0.48	0.083	3.356	0.28
	(0.003)	(0.422)		(0.004)	(0.652)		(0.006)	(0.655)	
8. Sugar	0.025	2.684	0.38	0.020	4.038	0.41	0.032	1.079	0.33
	(0.001)	(0.161)		(0.002)	(0.224)		(0.002)	(0.254)	
9. Tea	0.013	1.071	0.28	0.014	0.952	0.36	0.009	1.169	0.15
	(0.001)	(0.082)		(0.001)	(0.124)		(0.001)	(0.124)	
10. Housing	0.070	0.601	0.26	0.076	2.549	0.30	0.033	–0.071	0.07
	(0.004)	(0.431)		(0.005)	(0.736)		(0.005)	(0.473)	
11. Clothing	0.114	5.653	0.50	0.112	5.326	0.53	0.120	4.466	0.44
	(0.004)	(0.515)		(0.005)	(0.810)		(0.007)	(0.765)	
12. Other Food	0.033	2.625	0.16	0.032	1.066	0.16	0.047	2.914	0.28
	(0.003)	(0.288)		(0.003)	(0.445)		(0.004)	(0.403)	
13. Other Non-food	0.238	0	0.68	0.254	0	0.72	0.180	0	0.58
	(0.006)			(0.007)			(0.008)		
2* Log likelihood		–8731.8	.8		–4201.8			–3162.4	

Standard errors in brackets.

Note: These parameters are based on the standard identifying assumption that γ_i for (13) 'other non-food' is zero. The b_i are marginal budget shares, and γ_i, minimum consumption requirements, defined in the test.

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