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Estimation of Elasticities of Substitution for CES Production Functions using Data on Selected Manufacturing Industries in Pakistan

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Firm-level stochastic CES production functions are specified for large- and small-scale firms in twelve manufacturing industries in Pakistan. Assuming that firms within specified asset-size categories for which aggregative data are available have the same levels of productive inputs, the elasticities of substitution of labour for capital are estimated, using weighted least-squares regression. For large-scale firms, the estimated elasticities are generally not significantly different from one, whereas for small-scale firms the elasticities are significantly smaller than one but greater than zero. These results indicate that there may exist more possibilities for the substitution of labour for capital in manufacturing industries in Pakistan than were claimed by earlier researchers. This finding has important policy implications for Pakistan's economic development.

1. INTRODUCTION

The possibility of an efficient capital-labour substitution is crucial for th success of most fiscal, financial and technological policies that are designed t increase employment in developing countries through the adoption of labour intensive techniques of production. If such possibilities exist, then labour can b substituted for capital without necessarily resulting in a decline in output. This issu crucially depends on whether the elasticity of substitution is positive.

Most empirical studies in which the elasticity of substitution is estimate involve the use of aggregative survey data. In such studies, the total value added b firms within given asset-size categories is modelled in terms of the correspondin values of relevant inputs of production. In this study, we consider production func tions that are defined in terms of data on individual firms and estimate the elasticit of substitution, using official Pakistani data in aggregative form, under the assump tion that the firms in the productive process satisfy certain conditions. Details of th methodology involved are presented in Battese and Malik (1986).

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2. PAKISTANI DATA ON MANUFACTURING INDUSTRIES

Data on manufacturing industries in Pakistan are available from periodic censuses and surveys of large- and small-scale firms. All registered factories in Pakistan are considered large-scale firms, whereas unregistered factories are considered small-scale firms. Any factory employing ten or more workers is required to be registered under Section 2(j) or 5(i) of the Factories Act 1934. Section 2(j) covers those factories which employ twenty or more workers on any day during the year and use power in their operation, whereas Section 5(i) covers those factories in which a manufacturing process is carried on with or without power, and which employ ten or more workers for at least one day of the year [Government of Pakistan (1982a)].

The major source of data on different aspects of Pakistan's large-scale manufacturing firms is the irregularly published *Census of Manufacturing Industries*, which covers all firms registered under Sections 2(j) and 5(i) of the Factories Act 1934 and collect data on written-down values of fixed assets, gross value of production, industrial costs, value added, employment and wages. The census data, however, have several major defects: (i) there appears to be a serious undercoverage of the manufacturing firms involved; (ii) the data are not available on a yearly basis; and (iii) the definitions of some variables have changed over time.

Kemal (1976) adjusted the census data to obtain his so-called "consistent" time-series data for manufacturing firms within the large-scale industries of Pakistan. These adjusted data have serious limitations and have been the subject of severe criticism: Meekal (1982) and Norbye (1978a and 1978b).

In our empirical analyses, we use the original published data from the censuses of large-scale firms within selected manufacturing industries for the years 1969-70, 1970-71, 1975-76 and 1976-77: Government of Pakistan (1973; 1977; 1980 and 1982a). These are the four most recent years for which data are published. The data give total values of variables for firms within the different asset-size categories.

We also report analyses of data on small-scale and household manufacturing firms within selected industries in Pakistan. These data were obtained from a sample survey of firms within urban areas in 1976-77. The survey involved 405 firms which employed 7,500 workers: Government of Pakistan (1982b).

Twelve two-digit-level manufacturing industries are selected for consideration: All Industries,[1] Food, Textiles, Chemicals, Printing and Publishing, Leather and Leather Products, Non-metallic Mineral Products, Basic Metals, Metal Products, Electrical Machinery, Non-electrical Machinery, and Transport. These industries are considered because there are a reasonable number of published observations on them

[1] The "All Industries" category, as reported in the Census of Manufacturing Industries, presents the grand total of data on different items on the industries concerned.

for each census year and because most were considered in the three previous Pakistani studies, Kaži *et al.* (1976), Kemal (1981) and Kazmi (1981). The first two studies considered data for large-scale firms, whereas Kazmi involved analyses of data from small-scale firms obtained from the Survey in 1976-77. These studies used indirect forms of the constant- and variable-returns-to-scale CES and VES production functions, specified in terms of the aggregative data available for the different asset-size categories. In these three Pakistani studies, few significant estimates were obtained for elasticities of substitution and so it was concluded that there were very limited possibilities for the substitution of labour for capital in the manufacturing industries of Pakistan. However, we conduct a more careful analysis of the published data and consider an efficient estimation of elasticities of substitution for production functions, which are defined in terms of firm-level data.

3. ANALYSES WITH DATA FOR LARGE-SCALE FIRMS[2]

Data on totals of value added, employment cost, daily employment, book value of capital stocks at the beginning of the census years, and the number of reporting firms are available for different asset-size categories for the two largest provinces, Punjab and Sind. If the published data for a given asset-size category for an industry were such that employment cost was greater than the value added, then these data were excluded from the analysis, as were the observations that were considered clear outliers relative to others of similar asset sizes. Of the 688 observations in the published tables for the twelve industries in the four years, only six observations were excluded on the basis of these criteria. Table 1 presents the numbers of different asset-size categories for which data on large-scale firms are used in the subsequent analyses, listed by census year and province for the twelve industries.

Initially, the possibility that the two provinces, Punjab and Sind, have different elasticities of substitution is considered. The CES production function with returns-to-scale parameter, ν, is defined, for each province and census year, by

$$Y_{ij} = \gamma \left\{ \delta K_i^{-\rho} + (1-\delta) L_i^{-\rho} \right\}^{-\nu/\rho} e^{U_{ij}}, \quad \ldots \quad \ldots \quad (1)$$

$$j = 1, 2, \ldots, r_i, \quad i = 1, 2, \ldots, n,$$

where Y_{ij} represents the value added for the jth reporting firm in the ith asset-size category; K_i and L_i represent the amount of capital and labour employed by firms in the ith asset-size category;[3] the random errors, U_{ij}, $j = 1, 2, \ldots, r_i$, $i = 1, 2, \ldots, n$,

[2] For a definition of the different asset-size categories for large- and small-scale industries see Government of Pakistan (1982a and 1982b).

[3] It is assumed that the quantities of capital and labour for firms in a given asset-size category are the same. While this is likely to be only an approximation for any given empirical situation, it is a sufficient condition for the identification and estimation of elasticities of substitution for CES production functions, defined in terms of firm-level data [see Battese and Malik (1980)].

Table 1

Numbers of Asset-size Categories in Published Data on Large-scale Firms

Industry	1969-70		1970-71		1975-76		1976-77		Total
	Punjab	Sind	Punjab	Sind	Punjab	Sind	Punjab	Sind	
All Industries[1]	13	13	6	13	7	7	9	7	75
Food	12	11	6	12	7	7	8	7	70
Textiles	12	13	6	13	7	7	8	9	75
Chemicals	13	11	6	15	9	11	9	11	85
Printing	8	8	4	9	5	5	5	5	49
Leather	7	6	3	4	5	3	6	5	39
Mineral Products	8	8	4	7	5	3	4	4	43
Basic Metals	9	6	5	9	7	5	6	5	52
Metal Products	10	11	4	9	4	5	5	6	54
Electrical Mach.	9	8	5	7	6	6	5	7	53
Non-electrical Mach.	10	7	4	7	5	4	5	3	45
Transport	9	7	4	5	5	4	3	5	42

[1]The All Industries category is an aggregate of all the industries covered in the Census.

are assumed to be independent and identically distributed normal random variables with means zero and variances, σ_U^2; r_i represents the number of reporting firms in the ith asset-size category; and n represents the number of asset-size categories for the given industry in the given province and the census year involved (see Table 1).

Given that the product and factor markets are perfectly competitive, it follows that the elasticity of substitution for the CES production function (Equation 1), $\sigma \equiv (1 + \rho)^{-1}$, can be estimated from the following associated indirect form: see Behrman (1982, p. 161) and Battese and Malik (1986).

$$\log (\bar{Y}_i/L_i) = \beta_0 + \beta_1 \log w_i + \beta_2 \log L_i + \log \bar{V}_i, \quad \ldots \quad \ldots \quad (2)$$

$$i = 1, 2, \ldots, n$$

where $\beta_1 \equiv \nu(\nu + \rho)^{-1}$ and $\beta_2 \equiv \rho(\nu - 1)(\nu + \rho)^{-1} = (\nu - 1)(1 - \beta_1)$; $\bar{Y}_i$ represents the sample mean of the value added by the firms in the ith asset-size category; w_i represents the wage rate for labourers in firms within the ith asset-size category; and

$$\bar{V}_i \equiv \frac{1}{r_i} \sum_{j=1}^{r_i} V_{ij} \text{ in which } V_{ij} \equiv e^{U_{ij}}.$$

It is readily verified that the elasticity of substitution is expressed in terms of the coefficients of the logarithms of wages and labour in Equation (2) by

$$\beta_1 \quad = \quad (1 + \beta_2)\sigma \quad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (3)$$

Thus, if the constant-returns-to-scale CES production function applies (i.e. $\nu = 1$), then the coefficient of the logarithm of labour, β_2, is zero, and so the coefficient of the logarithm of wages, β_1, in the indirect form (Equation 2) is equal to the elasticity of substitution. Irrespective of the value of the returns-to-scale parameter, the CES production functions for the provinces of Punjab and Sind have the same elasticities of substitution if the coefficients of the logarithms of wages and labour are the same for the two provinces.

It follows from standard asymptotic methods that if the number of reporting firms within the *i*th asset-size category, r_i, is large enough, then the random variable, $\log \bar{V}_i$, in the indirect form (Equation 2) has approximately normal distribution with mean, $\frac{1}{2}\sigma_U^2$, and variance, $(e^{\sigma_U^2} - 1)/r_i$. Further, given appropriate regularity conditions,[4] it follows that consistent and asymptotically efficient estimators for the coefficients of the logarithms of wages and labour in the indirect form (Equation 2) are obtained by applying weighted least-squares regression to the indirect form (Equation 2), where the observations are weighted by the square roots of the numbers of reporting firms in the corresponding asset-size categories.

The test statistics involved for testing if the provinces of Punjab and Sind have the same CES production functions yielded non-significant values for most industries in the census years involved. There are indications that the constant-returns-to-scale CES production functions for the two provinces are not the same for All Industries (for 1975-76 and 1976-77), Textiles (for 1970-71), Chemicals (for 1975-76) and Non-electrical Machinery (for 1969-70). For the variable-returns-to-scale CES production function, the hypothesis of equal elasticities of substitution for the two provinces is rejected (at the five-percent level of significance) in only two cases, namely All Industries (for 1976-77) and Food (for 1975-76). Further, the hypothesis that the two provinces have the same CES production functions (i.e. the efficiency parameters, γ, are the same, as well as the elasticities of substitution) is only rejected in a few cases. Because of these results, the data for the two provinces are pooled and the analyses reported below assume that the two provinces have the same CES production functions for given industries.

Estimates for the elasticities of substitution for the twelve industries in each of the four census years are presented in Table 2, under the assumptions that the

[4]The number of firms, r_i, within the *i*th asset-size category and the number of asset-size categories, n, must approach infinity. Further, the matrix of transformed values of the independent variables in the indirect form (2) must be regular [see, e.g., Theil (1971, p. 363)].

Table 2

Estimates for Elasticities of Substitution, given that the Constant- or Variable-Returns-to-Scale CES Production Functions apply to Large-scale Firms

Industry	1969-70		1970-71		1975-76		1976-77	
	CRS[1]	VRS[2]	CRS	VRS	CRS	VRS	CRS	VRS
All Industries	1.62**	1.39*	1.38**	0.99**	0.92**	0.64**	1.74**	1.79**
	(0.48)	(0.66)	(0.17)	(0.13)	(0.23)	(0.19)	(0.10)	(0.16)
Food	1.41**	0.93	0.97**	0.51*	1.99**	2.67*	1.53**	1.32
	(0.44)	(0.55)	(0.30)	(0.22)	(0.63)	(1.28)	(0.13)	(0.23)
Textiles	0.12	0.28	0.76**	0.68**	1.49**	1.47**	1.72**	1.78**
	(0.51)	(0.42)	(0.23)	(0.16)	(0.15)	(0.15)	(0.16)	(0.18)
Chemicals	1.67**	1.44**	1.38**	1.91*	1.35**	0.90*	1.39**	1.58*
	(0.21)	(0.47)	(0.20)	(0.68)	(0.25)	(0.34)	(0.21)	(0.63)
Printing	1.35**	1.29*	0.98*	0.69*	1.36**	1.69**	1.33**	1.13*
	(0.27)	(0.46)	(0.35)	(0.27)	(0.17)	(0.25)	(0.32)	(0.40)
Leather	1.11	0.48	1.57*	0.98	0.98*	0.98*	1.72**	1.52**
	(0.99)	(0.71)	(0.70)	(0.65)	(0.44)	(0.48)	(0.54)	(0.54)
Mineral Products	1.70**	0.96*	1.32**	0.83*	1.82**	1.91*	0.96**	0.94*
	(0.38)	(0.41)	(0.23)	(0.30)	(0.28)	(0.64)	(0.18)	(0.24)

– Continued

Table 2 – (*Continued*)

Basic Metals	0.98*	0.85	0.85	0.33	1.71**	2.44**	1.83**	5.21
	(0.47)	(0.71)	(0.51)	(0.52)	(0.50)	(0.59)	(0.54)	(3.23)
Metal Products	1.10**	1.23**	1.35**	0.87**	1.19**	1.35*	1.51**	1.50**
	(0.24)	(0.35)	(0.18)	(0.16)	(0.10)	(0.37)	(0.18)	(0.52)
Electrical Mach.	1.00**	0.27	1.85**	2.42*	1.55**	2.16**	1.39**	1.86*
	(0.42)	(0.68)	(0.28)	(1.05)	(0.15)	(0.62)	(0.21)	(0.60)
Non-electrical	1.16**	0.74*	1.14**	0.90**	2.51**	5.74	1.37**	2.15*
Mach.	(0.34)	(0.29)	(0.29)	(0.24)	(0.62)	(2.88)	(0.31)	(0.81)
Transport	0.95**	1.19*	0.30	0.04	1.73**	1.50**	2.04**	2.05*
	(0.30)	(0.44)	(0.54)	(0.45)	(0.35)	(0.28)	(0.27)	(0.54)

[1] CRS denotes "constant returns to scale".
[2] VRS denotes "variable returns to scale".
*denotes "significant at the five-percent level".
**denotes "significant at the one-percent level".

constant- and variable-returns-to-scale CES production functions apply. Estimates for the standard errors of the elasticity estimators are presented in parentheses below the estimates. For the constant-returns-to-scale CES production function, the elasticity estimates are obtained by the weighted least-squares regression for the indirect form (Equation 2) in which the logarithm of labour is omitted from the model. For the variable-returns-to-scale CES production function, the elasticity estimates reported are the values of the estimator, $\hat{\sigma} = \hat{\beta}_1 (1 + \hat{\beta}_2)^{-1}$, where $\hat{\beta}_1$ and $\hat{\beta}_2$ are the weighted least-squares estimators for the coefficients of the logarithms of wages and labour in the indirect form (2). This is a consistent estimator for the elasticity, but it does not have a finite mean or variance. The estimated standard errors are the square roots of the asymptotic variance of the estimator, obtained by standard asymptotic methods.

Almost all the elasticity estimates for the constant-returns-to-scale CES production functions are significantly different from zero at the five percent level. In 1969-70, the elasticities for Textiles and Leather are not significant at the five-percent level, whereas the elasticity for Basic Metals is significant at the five-percent level and the other nine elasticities are significant at the one-percent level. In 1970-71, the elasticities for Basic Metals and Transport are not significant at the five-percent level, the elasticities for Printing and Leather are significant at the five-percent level, but for the remaining eight industries the elasticities are significant at the one-percent level. In 1975-76, the elasticity of substitution for Leather is significant at the five-percent level, but all other elasticities are significant at the one-percent level. In 1976-77, the elasticities for all twelve industries are significant at the one-percent level. For all twelve industries, the elasticity estimates in the four census years are such that at least three are significant at the five-percent level. Seven industries have elasticity estimates that are significant at the one-percent level for all four years. The significant elasticity estimates for the constant-returns-to-scale CES production function range from 0.92 to 2.51.

The elasticity estimates for the variable-returns-to-scale CES production function in Table 2 are generally quite similar to those for the constant-returns-to-scale CES production function. The estimated standard errors of the elasticity estimators for this production function are generally larger than for the constant-returns-to-scale model and so there are fewer significant estimates. However, with the variable-returns-to-scale CES production function, the elasticity estimates for all industries considered are significantly different from zero for at least one of the census years, and, in addition, almost eighty percent of the estimates are significant at the five-percent level.

Goodness-of-fit statistics for the CES production functions, defined by the squares of the estimated correlation coefficients between the observed logarithms of the value added per unit of labour and their predicted values, obtained by using the weighted least-squares estimates of the parameters of the indirect form (Equation 2) [see Battese and Griffiths 1980)], generally have values between 0.20 and 0.90. (These statistics are not reported here.) Although there are some cases where these goodness-of-fit statistics are quite low, their average values for given industries are between 0.35 and 0.77, the overall average being about 0.6.

It is evident from Table 2 that for any given industry the estimated elasticities of substitution vary somewhat over years. This raises the issue of whether the elasticity is constant over years. The values of test statistics involved in testing the hypotheses that the constant- and-variable-returns-to-scale CES production functions have the same elasticities for the four census years are presented in Table 3. The tests involve weighted least-squares estimation of the indirect forms (Equation 2) of the CES production functions, defined for the four census years, such that the intercept coefficients are not constrained to satisfy any particular relationship. Also presented in Table 3 are estimates of the elasticities of substitution, given that the elasticity is constant for the four census years.

The hypothesis that the four yearly CES production functions have the same elasticity of substitution is rejected for Textiles and Transport for both the constant- and variable-returns-to-scale models. Textiles is the only industry for which the elasticity estimates increase monotonically for the four years. For the variable-returns-to-scale CES production function, the hypothesis of constant elasticity over time is also rejected for Non-electrical Machinery. For all other industries, the assumption of a constant elasticity of substitution over time is accepted and so the pooling of the data for the four census years is reasonable to efficiently estimate the elasticities.[5] The estimated elasticities, obtained by pooling the data for the four census years, range from 1.20 to 1.59 for the constant-returns-to-scale CES production function and from 0.99 to 1.83 for the variable-returns-to-scale CES production function. All these elasticity estimates are significantly different from zero at the one percent level.

Since all the elasticity estimates in Table 3 (and almost all those in Table 2) are significantly greater than zero, it is of interest to test whether the elasticities are significantly different from one, which applies for the Cobb-Douglas production function. The elasticity of substitution is one if the relationship between the coefficients of the logarithms of wages and labour in the indirect form (2) is defined by $\beta_1 = (1 + \beta_2)$ [cf. Equation (3)]. Using traditional regression procedures for testing

[5]The hypothesis that the elasticities of the yearly CES production functions are constant over time and, in addition, the intercept coefficients of the indirect forms are a linear function of time was also considered. For the constant-returns-to-scale CES production function, the hypothesis is rejected for Textiles (at the one-percent level), Printing and Mineral Products (at the five-percent level). For the variable-returns-to-scale CES production function, the hypothesis is rejected for Textiles and Non-electrical Machinery (at the one-percent level). These analyses suggest that the use of aggregative time-series data may obtain spurious elasticity estimates for some industries [cf. Battese and Malik (1986)].

Table 3

Test Statistics for the Hypothesis that the Yearly Constant- and Variable-Returns-to-Scale CES Production Functions for Large-scale Firms have the Same Elasticity and the Estimates for that Elasticity

Industry	Constant Returns to Scale		Variable Returns to Scale	
	F[1]	σ	F[2]	σ
All Industries	2.47	1.31** (0.13)	1.63	1.02** (0.14)
Food	1.37	1.38** (0.20)	1.62	1.08** (0.25)
Textiles	6.15**	1.20** (0.14)	5.66**	1.13** (0.13)
Chemicals	0.52	1.46** (0.10)	0.75	1.37** (0.24)
Printing	0.44	1.24** (0.14)	1.06	1.07** (0.17)
Leather	0.24	1.36** (0.34)	0.48	1.05** (0.29)
Mineral Products	1.81	1.34** (0.15)	1.21	0.99** (0.17)
Basic Metals	1.03	1.35** (0.25)	1.71	1.52** (0.48)
Metal Products	0.65	1.25** (0.12)	1.17	1.17** (0.18)
Electrical Machinery	1.38	1.49** (0.14)	1.12	1.75** (0.39)
Non-electrical Machinery	2.80	1.59** (0.21)	6.97**	1.83** (0.35)
Transport	4.16*	1.36** (0.20)	2.70*	1.22** (0.24)

[1] These statistics are approximately $F_{3, n-6}$ random variables, given that the four yearly constant-returns-to-scale CES production functions have the same elasticity of substitution.

[2] These statistics are approximately $F_{6, n-12}$ random variables, given that the four yearly variable-returns-to-scale CES production functions have the same homogeneity and substitution parameters (and hence elasticity).

*denotes "significant at the five-percent level".

**denotes "significant at the one-percent level".

this hypothesis (or alternatively, using an asymptotic test on the estimated elasticity) it is found that the pooled elasticity estimates for All Industries, Mineral Products, Metal Products, Chemicals, Electrical Machinery and Non-electrical Machinery are significantly different from one (the first three at the five-percent level and the last three at the one percent level) for the constant-returns-to-scale CES production function. However, none of the pooled elasticity estimates are significantly different from one for the variable-returns-to-scale CES production function.

It is noted that the elasticity estimates presented in Tables 2 and 3 for the variable-returns-to-scale CES production function are generally smaller than for the constant-returns-to-scale CES production function. These elasticities are not significantly different if the coefficient of labour, β_2, in the indirect form (Equation 2) is zero. Given that the variable-returns-to-scale CES production function applies, the hypothesis, that the coefficient of labour is zero, is rejected for All Industries, Leather and Mineral Products (at the five-percent level of significance) amongst the nine industries for which the elasticity estimates appear to be constant for the four census years. For Textiles (in 1969-70 and 1970-71) and Non-electrical Machinery (in 1969-70 and 1975-76), the hypothesis, that the constant-returns-to-scale CES production function is adequate, is rejected at the five-percent level of significance.[6] These results suggest that for about half of the twelve industries considered, the elasticities of substitution are appropriately estimated using the constant-returns-to-scale CES production function. For the remaining industries, a more precise investigation of elasticities may require the consideration of the variable-returns-to-scale CES production function.

4. ANALYSES WITH DATA FOR SMALL-SCALE FIRMS

Data obtained from the 1976-77 survey of small-scale and household manufacturing firms (Government of Pakistan, 1982b) are published in aggregative form for specified asset-size categories, according to the same format as for the censuses of large-scale manufacturing firms. The number of different asset-size categories for which data are available on small-scale firms (not involving households) in the provinces of Punjab and Sind are presented in Table 4 for the twelve industries involved.

The test statistic for testing that the two provinces have the same elasticities of substitution has a significant value for All Industries only, irrespective of whether

[6]There seems to be no strong techno-economic reasoning for the finding that the constant-returns-to-scale CES production function seems to fit several industries but not textiles and non-electrical machinery. This is especially true since the constant-returns-to-scale CES production function appears to be adequate even in the case of textiles (in 1975-76 and 1976-77) and non-electrical machinery (in 1970-71 and 1976-77). This could have resulted from faulty data [see Kemal (1976)].

Table 4

Number of Asset-size Categories in published Data on Small-scale Firms

Industry	Punjab	Sind	Total
All Industries	7	7	14
Food	7	7	14
Textiles	6	5	11
Chemicals	11	7	18
Printing	6	5	11
Leather	5	4	9
Mineral Products	6	5	11
Basic Metals	6	5	11
Metal Products	7	7	14
Electrical Machinery	6	4	10
Non-electrical Machinery	6	5	11
Transport	5	5	10

the constant- or variable-returns-to-scale CES production function is assumed to apply. This implies that the data on the two provinces can be pooled to efficiently estimate the elasticity of substitution for each of the last eleven industries.

Estimates for the elasticities of substitution, obtained by pooling the data for both provinces, for all the twelve manufacturing industries are presented in Table 5, under the assumptions that the constant- and variable-returns-to-scale CES production functions apply. For the constant-returns-to-scale model, the elasticity estimates are significant at the one-percent level, except for Chemicals, Mineral Products, Metal Products and Transport. For the variable-returns-to-scale model, the majority of the industries have elasticity estimates which are not significant at the five-percent level.

These elasticity estimates are smaller than those obtained with the data for the large-scale firms for all twelve industries. Further, the estimates obtained with the data for the small-scale firms are significantly different from one. This could be due to the narrower asset-size categories for the small-scale data, which restricts the range of techniques involved.

A comparison of the elasticity estimates presented in Table 5 indicates that the estimates for the variable-returns-to-scale CES production function are generally smaller than for the constant-returns-to-scale model, although it appears that, except for Food, the estimates from the two models are not significantly different. In fact, the hypothesis that the coefficient of the logarithm of labour, β_2, in the indirect form (Equation 2) is zero is only rejected for Food, Mineral Products and Transport

Table 5

Estimates for Elasticities of Substitution, given that the Constant- or Variable-Returns-to-Scale CES Production Functions apply to Small-scale Firms

Industry	1976-77	
	CRS[1]	VRS[2]
All Industries	0.44** (0.05)	0.35** (0.07)
Food	0.53** (0.12)	0.19** (0.06)
Textiles	1.21** (0.10)	1.52** (0.46)
Chemicals	0.16 (0.18)	0.09 (0.21)
Printing	0.88** (0.23)	0.38 (0.29)
Leather	1.40** (0.33)	0.87 (1.07)
Mineral Products	0.15 (0.33)	0.15 (0.16)
Basic Metals	0.29** (0.11)	0.22 (0.26)
Metal Products	0.25 (0.13)	0.21 (0.29)
Electrical Machinery	0.55** (0.11)	0.58** (0.19)
Non-electrical Machinery	0.28** (0.08)	0.09 (0.10)
Transport	0.11 (0.10)	0.47 (0.25)

[1] CRS denotes "constant returns to scale". The figures in parentheses are estimated standard errors.

[2] VRS denotes "variable returns to scale".

*denotes "significant at the five-percent level".

**denotes "significant at the one-percent level".

(at the one percent level). For the remaining nine industries, the constant-returns-to-scale CES production function is an adequate representation for the data for small-scale firms, given that the specifications of the variable-returns-to-scale CES model apply.

5. COMPARISON WITH OTHER STUDIES

It is difficult to compare estimates of the elasticity of substitution across studies, because of the possible differences in definitions of variables, methods used and the time periods to which they relate. However, we compare the results obtained in this study with those obtained in the Pakistani studies by Kazi *et al.* (1976) and Kazmi (1981) using data from large- and small-scale firms, respectively. The estimates obtained in the three studies are presented in Table 6.

The estimates reported by Kazi *et al.* (1976) are obtained by using ordinary least-squares regression for nine of the twelve industries considered in this study. Five of these industries are reported to have elasticity estimates that are significantly different from zero. The elasticity estimate reported by Kazi *et al.* (1976) for food is negative. This is likely to be due to data problems. The reported value-added for the smallest asset-size category is relatively high, whereas the wages are extremely low. This may be due to the prevalence of unpaid family labour and the production of outputs that command premium prices due to traditional preferences.

Of the elasticity estimates reported from Kazmi (1981) for six of the twelve industries considered in this study, only two are reported to be significant at the five-percent level. In our analyses for these six industries, five have elasticity estimates that are significant at the one percent level.

We believe that a careful examination of the data for different asset-size categories and the use of weighted least-squares regression, associated with the estimable indirect forms of the firm-level production functions, yield more precise (and hence significant) estimates for the elasticities of substitution. The results obtained by these methods suggest that there exist more possibilities for the substitution of labour for capital in two-digit-level manufacturing industries in Pakistan than was previously considered.

6. CONCLUSIONS

The analyses conducted for both large- and small-scale firms within the selected manufacturing industries in Pakistan indicate that constant-returns-to-scale exist for the majority of industries for which the specifications of the CES production function apply. Given our modelling for firm-level data, and the consequent efficient method of estimation of the elasticities of substitution, it appears that almost all the

Table 6

Comparison of Estimates of the Elasticity of Substitution, based upon the Constant-Returns-to-Scale CES Production Function for Different Studies in Pakistan

Industry	Large-scale Firms, 1969-70		Small-scale Firms, 1976-77	
	Kazi *et al.* (1976)	This Study	Kazmi (1981)	This Study
All Industries	1.17*	1.62** (0.48)	0.47*	0.44** (0.05)
Food	–0.30	1.41** (0.44)	0.58	0.53** (0.12)
Textiles	0.18	0.12 (0.51)	0.08	1.21** (0.10)
Chemicals	1.86**	1.67** (0.21)	0.51	0.16 (0.18)
Printing	1.73**	1.35** (0.27)	0.37	0.88** (0.28)
Leather	0.46	1.11 (0.99)	1.88*	1.40** (0.33)
Mineral Products	1.64**	1.70** (0.38)	–	0.15 (0.33)
Basic Products	1.29**	0.98** (0.47)	–	0.29** (0.11)
Metal Products	–	1.10** (0.24)	–	0.25 (0.13)
Electrical Mach.	0.81	1.00** (0.42)	–	0.55** (0.11)
Non-electrical Mach.	–	1.16** (0.34)	–	0.28** (0.08)
Transport	–	0.95** (0.30)	–	0.11 (0.10)

*denotes "significant at the five-percent level".
**denotes "significant at the one-percent level".
–denotes "not available".

industries considered have elasticities significantly greater than zero. For large-scale firms, the elasticities are generally not significantly different from one, whereas for the small-scale firms the elasticities are significantly less than one. Although this implies that the CES production functions for large-scale firms are not significantly different from Cobb-Douglas production functions, the use of the indirect forms of the CES production functions to estimate the elasticities of substitution circumvent the need to use capital data which are likely to be quite unreliable for developing countries.

However, given that any two-digit manufacturing industry is a heterogeneous group for which many firms produce a large variety of outputs, it is difficult to determine the extent to which a given positive estimate for the elasticity of substitution indicates the possibilities for substitution of labour for capital in the efficient production of a given homogeneous output. Given this, and the stringent conditions under which the elasticity of substitution is indentified and estimable from aggregate data, there is a need for data analysis at more disaggregated levels. Even if different industries have the same elasticities of substitution, it is preferable to conduct analyses on the different industries and obtain a pooled estimate for the common elasticity. This will result in better precision of estimation than aggregating the data prior to estimation. For example, if the last eleven industries (Food, Textiles, etc.) have CES production functions which have the same elasticities for the four census years considered, then the pooled elasticity estimated from the data in Table 3 is 1.36, with an estimated standard error of 0.05. This compares with the elasticity estimate for All Industries of 1.31, which has an estimated standard error of 0.13. However, the latter estimate is obtained under the assumption that the inputs for firms within the different asset-size categories are the same. This is almost certainly false for the aggregation of data for the individual manufacturing industries in Pakistan.

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