

# Dynamic Stability, Wage Subsidies and the Generalized Harris-Todaro Model

M. ALI KHAN\*

In a recent contribution, Neary has shown that paradoxes and instability correspond in the two-sector model with proportional differentials in factor returns, leading one to downgrade the importance of these paradoxes. In this paper, we examine the extent to which this result extends to the Generalized Harris-Todaro Model of which the proportional differential setting is a simple special case. In developing the argument, we generate a variety of comparative-statics results of consequence for development theory. The implications of these results for the conduct of commercial policy are also brought out.

## 1. INTRODUCTION

It is by now well known that in the presence of factor-market distortions, a sector may be more capital-intensive in *physical* terms and less capital-intensive in *value* terms and that this leads to a theory riddled with paradoxes; see Magee [16] or Hazari [11] for a comprehensive treatment. In more specific terms, Rybczynski's Theorem is dependent on

$$\text{Sign } [k_u(\omega_u) - k_r(\omega_r)] \quad (1.1)$$

whereas the Stolper-Samuelson Theorem relies on

$$\text{Sign } [\theta_{uK}/\theta_{uL} - \theta_{rK}/\theta_{rL}] = \text{Sign } [\theta_{rL}\theta_{uK} - \theta_{rK}\theta_{uL}] \quad (1.2)$$

where  $k_i$ ,  $\omega_i$  and  $\theta_{iK}/\theta_{iL}$  are respectively the capital-labour ratio, the wage-rental ratio and the ratio of factor shares, all for sector  $i$ ,  $i = u, r$ . When equation (1.1) conflicts with equation (1.2), we obtain a lack of correspondence between the two theorems and also perverse price-output and distortion-output responses.

In a recent contribution, Neary [17] has put forward an adjustment process under which an equilibrium is locally asymptotically stable<sup>1</sup> when equation (1.1) is in agreement with equation (1.2) and unstable otherwise. This leads Neary to the

\*The author is Associate Professor of Economics at the Johns Hopkins University, Baltimore, Md. (U.S.A.). This paper was presented at the Trade and Development Seminar at Columbia University and at the University of California at Davis. He thanks Guillermo Calvo, Ron Findlay, Lou Maccini, Bob Mundell and Leon Wegge for their comments and helpful suggestions. The influence of Terence Gorman on Section 6 should be apparent. Errors are, of course, solely the author's.

<sup>1</sup>See Hirsch and Smale [12, p. 186] for a precise definition.



interesting conclusion that "all these paradoxes are theoretical curiosa which will 'almost never' be observed in real world economies." The principal object of this paper is to examine the relevance of this insight to the study of the effects of wage subsidies on urban output and employment. This is a problem of some importance for development theory and policy and one to which counterintuitive answers are easily obtained as was first pointed out in a pioneering paper by Corden and Findlay [8].

Corden and Findlay studied a two-sector, small open economy in which capital is mobile but labour markets equilibrate in accordance with the Harris-Todaro [10] hypotheses, viz. a rigid urban wage and equality of expected<sup>2</sup> nominal wages; see also Stiglitz [21]. They showed that in such an economy, urban output and employment could rise if the urban wage is increased. They wrote, "Contrary to one's intuition it can be shown that such a paradoxical outcome is actually possible on not too implausible assumptions. A sufficient condition is simply that there are fixed coefficients of production in both sectors with the urban sector being relatively more capital-intensive. The paradox would still follow if some limited degree of technical substitution were possible."

In Khan [15], a generalized Harris-Todaro model is presented in which the urban wage is a function of the rural wage, the urban unemployment rate and the rental; this dependence being quantified by the respective elasticities  $e_w$ ,  $e_\lambda$  and  $e_R$ . Specific values of these elasticities not only yield the Corden-Findlay model but also allow us to incorporate considerations arising from labour turnover as in Stiglitz [19], or the efficiency wage, as in Stiglitz [20], or the presence of trade unions, as in Calvo [6], or from costly supervision, as in Calvo and Wellisz (see Calvo [5]).<sup>3</sup> The model also yields the traditional absolute or proportional wage-differential model as a special case. This generalized model exhibits

- (i) a lack of correspondence between the Rybczynski and Stolper-Samuelson Theorems, and
  - (ii) a perverse distortion-unemployment rate<sup>4</sup> response,
- provided that suitable generalizations of the physical and value factor intensities lead to conflicting rankings of the two sectors. Specifically, equation (1.1) has to be replaced by *unemployment-adjusted* factor intensities, i.e.

$$\text{Sign} [k_u (\omega_u)/(1+\lambda) - k_r (\omega_r)] = \text{Sign} [\theta_{rL} \theta_{uK} - \theta_{rK} \theta_{uL}] \quad (1.3)$$

<sup>2</sup>The expected wage is the wage times the probability of getting a job, this probability being proxied by the unemployment rate.

<sup>3</sup>It may be worth pointing out that the discussion in Khan [15] makes no mention of either the efficiency wage or the costly supervision hypothesis.

<sup>4</sup>The fact that it is the unemployment rate rather than the size of the unemployment pool deserves emphasis.

and equation (1.2) by the *elasticities-adjusted* factor intensities, i.e.

$$\text{Sign} [\theta_{rL} (\theta_{uK} (1 - e_\lambda) + \theta_{uL} e_R) - \theta_{rK} \theta_{uL} (e_w - e_\lambda)] \quad (1.4)$$

When  $e_w$  is unity and  $e_\lambda$ ,  $e_R$  and the urban rate of unemployment,  $\lambda$ , are all zero, as in the standard case of a proportional wage differential, equations (1.3) and (1.4) become identical to equations (1.1) and (1.2) respectively.

Given the variety of labour market conditions embraced by the model, a systematic treatment of the effect of wage subsidies and income taxes on urban output and employment reduces to a study of

- (iii) price-output/employment responses,
- (iv) differential-output/employment responses, and
- (v) distortion-output/employment responses.

A change in the differential is an exogenous change in the spread between the rural and the urban wages and a change in the distortion is an exogenous shift in the function determining the urban wage.

The fact that the generalized Harris-Todaro model will exhibit perverse responses is obvious; what is interesting in the light of Neary's work is answers to the following questions:

- (a) Do (iii), (iv), and (v) hinge, like (i) and (ii), on the agreement or conflict of equations (1.3) and (1.4)?
- (b) Does there exist an adjustment mechanism under which an equilibrium is locally asymptotically stable when equation (1.3) is in agreement with equation (1.4), and unstable otherwise?

We obtain an affirmative answer to (b) and a mixed answer to (a). These results are satisfying from the viewpoint of both policy and theoretical analysis and also have implications for normative economics. The effect of urban wage subsidies and taxes on urban employment and output is of interest to policy makers in developing countries and the results delineate circumstances when they can and cannot ignore perverse responses. In terms of analysis, note that the results give further support to the viewpoint emphasized in Khan [15] that the Harris-Todaro literature should be seen in the context of the theory of proportional wage differentials.<sup>5</sup>

For normative economics we have to turn to the work of Bhagwati and Srinivasan [3 and 18] who have comprehensively studied welfare theory of second-best policy interventions in a "small" as well as a "large" open economy. In particular, they have shown that for both a "small" and a "large" economy, "A wage subsidy (in manufacturing) will exist which will improve welfare over *laissez faire*." However, Bhagwati and Srinivasan confine themselves to a world with

<sup>5</sup>For a less than full subscription to this point of view, see the first paragraph of Corden and Findlay's paper [8].



immobile capital and a rigid urban wage, and a natural question arises as to the conditions under which their proposition is valid with intersectoral capital mobility and in the variety of labour market settings as are implicit in the generalized Harris-Todaro model. Our results, along with the notion of an expenditure function, allow one to answer this question rather easily; the ease and economy offered by the duality approach also suggest wider applicability.

The paper proceeds as follows. Section 2 is devoted to a brief and stark presentation of the model and some preliminary analysis. Section 3 presents the various paradoxes in the abstract setting of the model in its full generality. Section 4 considers implications of the results of Section 3 for the effect of wage subsidies in differing labour market conditions. Section 5 presents the adjustment process and Section 6 is devoted to the Bhagwati-Srinivasan proposition. The last section of the paper is devoted to some concluding remarks.

## 2. THE MODEL AND PRELIMINARY ANALYSIS<sup>6</sup>

Let a country consist of an urban and a rural sector, indexed by  $u$  and  $r$  respectively, and be endowed with non-negative amounts of labour  $\mathcal{L}$  and capital  $\kappa$ . Let the  $i^{\text{th}}$  sector produce a commodity in amount  $X_i$  in accordance with a production function

$$X_i = F_i(L_i, K_i) \quad i = u \text{ and } r \quad (2.1)$$

which is assumed to be positively homogeneous of degree 1, twice continuously differentiable and with its intensive form  $f_i(k_i)$ ,  $k_i = (K_i/L_i)$ , having its first derivative everywhere positive and second derivative everywhere negative. The  $L_i$  and  $K_i$  are allocations of labour and capital and are determined through marginal productivity pricing. We thus have

$$p_r \partial F_r / \partial K_r = p_r f'_r(k_r) = R = p_u \partial F_u / \partial K_u = p_u f'_u(k_u) \quad (2.2a)$$

$$p_r \partial F_r / \partial L_r = p_r(f_r(k_r) - k_r f'_r(k_r)) = w_r \quad (2.2b)$$

$$p_u \partial F_u / \partial L_u = p_u(f_u(k_u) - k_u f'_u(k_u)) = w_u \quad (2.2c)$$

The country is too small to influence  $p_u$  and  $p_r$ , positive international prices of the two commodities.

The equilibrium in the labour market is given by

$$w_u = \tau(1+\lambda)w_r \quad (2.3)$$

<sup>6</sup>The model, more specifically equations (2.1) to (2.10), are reproduced from Khan [15]. The analysis, of course, heavily relies on the treatment found in Jones [13; 14].

where  $\tau$  is a shift parameter and  $\lambda$  is the ratio of the unemployed to the urban employed. Thus  $L_u/L_u(1+\lambda)$  can be taken to be the probability of finding a job in the urban sector, a formalization due to Harris and Todaro [10]. We shall assume that the urban wage is endogenously determined and that this endogeneity is brought out by

$$w_u = \Omega(w_r, \lambda, R, \Upsilon) \quad (2.4)$$

where  $\Upsilon$  is a shift parameter. For a discussion of the microfoundations of  $\Omega(\cdot)$ , see Khan [15] as well as Table 1 and Section 4 below. For the immediate discussion, all we need are the following elasticities:

$$e_w = \frac{\partial \log \Omega(\cdot)}{\partial \log w_r}, e_j = \frac{\partial \log \Omega(\cdot)}{\partial \log j}, j = \lambda, R \text{ and } \Upsilon \quad (2.5)$$

Table 1

*The Microfoundations of the Function  $\Omega(w_r, \lambda, R, \Upsilon)$*

Labour-Market Conditions	Elasticities of $\Omega(\cdot)$		
	$e_w$	$e_\lambda$	$e_R$
1. Identical Wages <sup>1</sup>	1	0	0
2. Proportional-Differential in Wages	1	0	0
3. Absolute Differential in Wages	+	0	0
4. Rigid Urban Wage	0	0	0
5. Efficiency Wage	0	0	0
6. Labour Turnover	0	—	0
7. Trade Unions	+	0	?
8. Costly Supervision	+	0	0

<sup>1</sup>This is, of course, the Heckscher-Ohlin-Samuelson setting.

Addition of the following two equations completes the specification of the model.

$$K_r + K_u = \kappa \text{ and } L_r + L_u(1+\lambda) = \mathcal{L} \quad (2.6)$$

Given constant returns to scale and absence of joint production, we can write down the cost functions in each sector as

$$p_i = C_i(w_i, R) \quad i = u \text{ and } r \quad (2.7)$$



If we append to equation (2.7) the equations (2.3) and (2.4), we have a model in reduced form with four equations for four unknowns. Denoting  $dx/x$  as  $\hat{x}$ , we obtain

$$\begin{bmatrix} (\theta_{uK} + \theta_{uL}e_R) & \theta_{uL}e_w & \theta_{uL}e_\lambda \\ \theta_{rK} & \theta_{rL} & 0 \\ -e_R & 1-e_w & 1-e_\lambda \end{bmatrix} \begin{bmatrix} \hat{R} \\ \hat{w}_r \\ \hat{\lambda} \end{bmatrix} = \begin{bmatrix} \hat{p}_u - \theta_{uL}e_\gamma \hat{\gamma} \\ \hat{p}_r \\ e_\gamma \hat{\gamma} - \hat{\tau} \end{bmatrix} \quad (2.8)$$

where  $\theta_{ij}$  is the share of factor  $j$  in the  $i^{\text{th}}$  sector. It can be checked by straightforward calculations that equation (2.8) yields

$$\begin{bmatrix} \hat{R} \\ \hat{w}_r \\ \hat{\lambda} \end{bmatrix} = \frac{1}{|\Delta|} \begin{bmatrix} \theta_{rL}(1-e_\lambda) & \theta_{uL}(e_\lambda - e_w) & -\theta_{uL}\theta_{rL}e_\lambda \\ -\theta_{rK}(1-e_\lambda) & \theta_{uK}(1-e_\lambda) + \theta_{uL}e_R & \theta_{rK}\theta_{uL}e_\lambda \\ \theta_{rK}(1-e_w) + \theta_{rL}e_R & -(\theta_{uL}e_R + \theta_{uK}(1-e_w)) & (\theta_{uK} + \theta_{uL}e_R)\theta_{rL} - \theta_{rK}\theta_{uL}e_w \end{bmatrix} \begin{bmatrix} \hat{p}_u - \theta_{uL}e_\gamma \hat{\gamma} \\ \hat{p}_r \\ e_\gamma \hat{\gamma} - \hat{\tau} \end{bmatrix} \quad (2.9)$$

where

$$|\Delta| = \theta_{rL}(\theta_{uK}(1-e_\lambda) + \theta_{uL}e_R) + \theta_{rK}\theta_{uL}(e_\lambda - e_w) \quad (2.10)$$

Equation (2.6) can be rewritten as

$$a_{rL}X_r + a_{uL}(1+\lambda)X_u = \mathcal{L} \text{ and } a_{rK}X_r + a_{uK}X_u = \kappa \quad (2.11)$$

where  $a_{iL} = L_i/X_i$  and  $a_{iK} = K_i/X_i$  ( $i = u, r$ ). Routine differentiation of equation (2.11) yields

$$\begin{bmatrix} \kappa_u & \kappa_r \\ \ell_u(1+\lambda) & \ell_r \end{bmatrix} \begin{bmatrix} \hat{X}_u \\ \hat{X}_r \end{bmatrix} = \begin{bmatrix} \hat{\kappa} - \kappa_r \hat{a}_{rK} - \kappa_u \hat{a}_{uK} \\ \hat{\mathcal{L}} - \ell_r \hat{a}_{rL} - \ell_u(1+\lambda) \hat{a}_{uL} - \ell_u(1+\lambda) \hat{\lambda} \end{bmatrix} \quad (2.12)$$

where  $\ell_i = L_i/\mathcal{L}$  and  $\kappa_i = K_i/\kappa$ ,  $i = u, r$ . Given the definition of the elasticity of substitution  $\sigma_i$  where

$$\hat{a}_{iK} - \hat{a}_{iL} = \sigma_i (\hat{w}_i - \hat{R}) \quad (i = u, r) \quad (2.13a)$$

and the identity

$$\hat{a}_{iK}\theta_{iK} + \hat{a}_{iL}\theta_{iL} = 0 \quad (i = u, r) \quad (2.13b)$$

we can rewrite equation (2.12) as

$$\begin{bmatrix} \kappa_u & \kappa_r \\ \ell_u(1+\lambda) & \ell_r \end{bmatrix} \begin{bmatrix} \hat{X}_u \\ \hat{X}_r \end{bmatrix} = \begin{bmatrix} \hat{\kappa} - \kappa_r \sigma_r \theta_{rL} (\hat{w}_r - \hat{R}) - \kappa_u \sigma_u \theta_{uL} (\hat{w}_u - \hat{R}) \\ \hat{\mathcal{L}} + \ell_r \sigma_r \theta_{rK} (\hat{w}_r - \hat{R}) + \ell_u (1+\lambda) \sigma_u \theta_{uK} (\hat{w}_u - \hat{R}) - \ell_u (1+\lambda) \hat{\lambda} \end{bmatrix} \quad (2.14)$$

Similarly, differentiation of equation (2.6) and substitution of the definition of  $\sigma_i$ , yields

$$\begin{bmatrix} \kappa_u & \kappa_r \\ \ell_u(1+\lambda) & \ell_r \end{bmatrix} \begin{bmatrix} \hat{L}_u \\ \hat{L}_r \end{bmatrix} = \begin{bmatrix} \hat{\kappa} - \kappa_r \sigma_r (\hat{w}_r - \hat{R}) - \kappa_u \sigma_u (\hat{w}_u - \hat{R}) \\ \hat{\mathcal{L}} - \ell_u (1+\lambda) \hat{\lambda} \end{bmatrix} \quad (2.15)$$

Equations (2.9), (2.14), and (2.15) will be the basis of the analysis to follow in the subsequent section.

### 3. THE PARADOXES

In the sequel we shall be relying on the following *Standing Hypothesis*:

$$0 \leq e_w \leq 1; e_\lambda \leq 0; 0 \leq e_R \leq 1 \text{ and } e_\gamma > 0.$$

For a justification of this, see Table 1 and Section 4 below.

We shall now take in turn each of the paradoxes referred to in the introduction.

#### 3.1. Lack of Correspondence Between Rybczynski and Stolper-Samuelson Theorems

From equation (2.14), it can be easily seen that Rybczynski's Theorem relies on the sign of  $|\Delta| \equiv \theta_{rL}\theta_{uK} - \theta_{uL}\theta_{rK}$  since



$$\text{Sign } |D| = \text{Sign} \left( \frac{k_u}{1+\lambda} - k_r \right) = \text{Sign} [\kappa_u \ell_r - \ell_u (1+\lambda) \kappa_r] \quad (3.1.1)$$

From equation (2.9) and the Standing Hypothesis, it can be seen that the Stolper-Samuelson Theorem relies on the sign of equation (2.10).

### 3.2. Perverse Distortion-Unemployment Rate Response<sup>7</sup>

It can be easily checked from equation (2.9) that

$$\hat{\lambda}/\hat{\Upsilon} = e_{\Upsilon}(\theta_{uK}\theta_{rL} - \theta_{uL}\theta_{rK})/|\Delta| \quad (3.2.1)$$

Thus,  $\text{Sign}(\hat{\lambda}/\hat{\Upsilon})$  depends on  $\text{Sign}(|D|/|\Delta|)$ . Note that this result relies on the Standing Hypothesis only to the extent that it bears on  $e_{\Upsilon}$ .

### 3.3. Perverse Price-Output/Employment Responses<sup>8</sup>

From equation (2.9) it can be checked that

$$\hat{w}_r - \hat{R} = -[(1-e_{\lambda})/|\Delta|] \hat{p}_u \quad (3.3.1a)$$

$$\hat{w}_u - \hat{R} = [(e_{\lambda} - e_w \theta_{rK} + \theta_{rL}(e_R - 1))/|\Delta|] \hat{p}_u \quad (3.3.1b)$$

$$\hat{\lambda} = [(\theta_{rK}(1-e_w) + \theta_{rL}e_R)/|\Delta|] \hat{p}_u \quad (3.3.1c)$$

The standing Hypothesis and the substitution of these formulae in equations (2.14) and (2.15) yield

$$\text{Sign}(\hat{X}_u/\hat{p}_u) = \text{Sign}(\hat{L}_u/\hat{p}_u) = \text{Sign}(|\Delta| \times |D|) \quad (3.3.2a)$$

$$\text{Sign}(\hat{X}_r/\hat{p}_u) = \text{Sign}(\hat{L}_r/\hat{p}_u) = -\text{Sign}(|\Delta| \times |D|) \quad (3.3.2b)$$

The effect of changes in  $\hat{p}_r$  on  $\hat{X}_i$  and  $\hat{L}_i$ , ( $i=u, r$ ), can be worked out to give analogous conclusions.

<sup>7</sup>In what follows, the shift parameter  $\Upsilon$  will be referred to as the *distortion* and the shift parameter  $\tau$  as the *differential*.

<sup>8</sup>As is well known, the existence of perverse price-output responses was first pointed out by Bhagwati and Srinivasan [4].

### 3.4. Perverse Differential-Output/Employment Responses

From equation (2.9) it can be checked that

$$(\hat{w}_r - \hat{R}) = -[e_{\lambda} \theta_{uL}/|\Delta|] \hat{\tau} \quad (3.4.1a)$$

$$(\hat{w}_u - \hat{R}) = -[e_{\lambda} \theta_{rL}/|\Delta|] \hat{\tau} \quad (3.4.1b)$$

$$\hat{\lambda} = -[\{(\theta_{uK}\theta_{rL} - \theta_{rK}\theta_{uL}) + (\theta_{rK}\theta_{uL}(1-e_w) + \theta_{uL}\theta_{rL}e_R)\}/|\Delta|] \hat{\tau} \quad (3.4.1c)$$

We then obtain from equations (2.14) and (2.15)

$$\left(\frac{\hat{X}_u}{\hat{\tau}}\right) = \frac{-w_u}{|\Delta||\Delta|} \left[ e_{\lambda} (\ell_r \xi + \kappa_r \xi) - \eta - \kappa_r \ell_u (1+\lambda)(\theta_{uK}\theta_{rL} - \theta_{rK}\theta_{uL}) \right] \quad (3.4.2)$$

$$\left(\frac{\hat{L}_u}{\hat{\tau}}\right) = \frac{-w_u}{|\Delta||\Delta|} \left[ e_{\lambda} \ell_r \Phi - \eta - \kappa_r \ell_u (1+\lambda)(\theta_{uK}\theta_{rL} - \theta_{rK}\theta_{uL}) \right] \quad (3.4.3)$$

where

$$\xi = (\kappa_r \sigma_r + \kappa_u \sigma_u) \theta_{rL} \theta_{uL} \geq 0 \quad (3.4.4a)$$

$$\phi = (\kappa_r \sigma_r \theta_{uL} + \kappa_u \sigma_u \theta_{rL}) \geq 0 \quad (3.4.4b)$$

$$\xi = (\ell_r \sigma_r \theta_{rK} \theta_{uL} + \ell_u (1+\lambda) \sigma_u \theta_{uK} \theta_{rL}) \geq 0 \quad (3.4.4c)$$

$$\eta = \kappa_r \ell_u (1+\lambda) (\theta_{rK} \theta_{uL} (e_R + (1-e_w))) \geq 0 \quad (3.4.4d)$$

We can similarly calculate  $(\hat{X}_r/\hat{\tau})$  and  $(\hat{L}_r/\hat{\tau})$ . The qualitative properties are shown in Table 2.

### 3.5. Perverse Distortion-Output/Employment Responses

Falling back again on equation (2.9) we obtain

$$\hat{w}_r - \hat{R} = [e_{\Upsilon} \theta_{uL}/|\Delta|] \hat{\Upsilon} \quad (3.5.1a)$$

$$\hat{\lambda} = [e_{\Upsilon}(\theta_{uK}\theta_{rL} - \theta_{uL}\theta_{rK})/|\Delta|] \hat{\Upsilon} \quad (3.5.1b)$$

$$\hat{w}_u - \hat{R} = [e_w \hat{w}_r + e_{\lambda} \hat{\lambda} + e_R \hat{R} - \hat{R} + e_{\Upsilon} \hat{\Upsilon}] = [e_{\Upsilon} \theta_{rL}/|\Delta|] \hat{\Upsilon} \quad (3.5.1c)$$

Substitution of equation (3.4) in equations (2.14) and (2.15) yields



$$\left(\frac{\hat{X}_u}{\hat{T}}\right) = -\frac{e_T}{|D||\Delta|} \left[ (\ell_r \xi + \kappa_r \xi) - \kappa_r \ell_u (1+\lambda) (\theta_{uK} \theta_{rL} - \theta_{rK} \theta_{uL}) \right] \quad (3.5.2)$$

$$\left(\frac{\hat{L}_u}{\hat{T}}\right) = -\frac{e_T}{|D||\Delta|} \left[ (\ell_r \phi - \kappa_r \ell_u (1+\lambda) (\theta_{uK} \theta_{rL} - \theta_{rK} \theta_{uL})) \right] \quad (3.5.3)$$

We can similarly calculate  $(\hat{X}_r/\hat{T})$  and  $(\hat{L}_r/\hat{T})$  and collect the qualitative properties in Table 2.

Table 2

*Differential-Output/Employment and Distortion-  
Output/Employment Responses*

		( D  x  \Delta ) > 0				( D  x  \Delta ) < 0			
		D <0,  \Delta <0		D >0,  \Delta >0		D <0,  \Delta >0		D >0,  \Delta <0	
$X_u/\hat{T}$	$(\hat{X}_r/\hat{T})$	?	(?)	—	(+)	?	(?)	+	(—)
$\hat{L}_u/\hat{T}$	$(\hat{L}_r/\hat{T})$	?	(?)	—	(+)	?	(?)	+	(—)
$\hat{X}_u/\hat{T}$	$(\hat{X}_r/\hat{T})$	—	(+)	?	(?)	+	(—)	?	(?)
$\hat{L}_u/\hat{T}$	$(\hat{L}_r/\hat{T})$	—	(+)	?	(?)	+	(—)	?	(?)

We can conclude this section with the finding that conflict of the signs of  $|\Delta|$  and  $|D|$  is necessary and sufficient for Sections (3.1), (3.2), and (3.3) but neither necessary nor sufficient for Sections (3.4) and (3.5).

#### 4. THE EFFECT OF WAGE SUBSIDIES ON URBAN OUTPUT AND EMPLOYMENT

In this section we shall use the preceding results to give conditions under which the effects of wage subsidies on output and employment depend on the agreement or conflict of the signs of  $|D|$  and  $|\Delta|$ . We shall consider both *ad valorem* and specific wage subsidies as well as urban income taxes and denote them by  $S_i$ ,  $V_i$  and  $T_u$  respectively,  $i = u, r$ . For subsequent notational convenience, let  $s_i = (1 - S_i)$ ,  $v_i = -V_i$  and  $t_u = 1 - T_u$ . The wage paid out in the  $i^{\text{th}}$  sector is given by  $w_i s_i + v_i$  and that received by urban workers is  $t_u w_u$ . In the results reported below, we shall be considering changes in  $s_i$ ,  $v_i$  and  $t_u$  and the reader should be mindful of the fact that the consequent decreasing of subsidies is in keeping with the Corden-Findlay experiment of an increased urban wage.

A wage subsidy and an urban income tax have two effects at the same time. Firstly, both shift the  $\Omega(\cdot)$  function in a manner that depends on the microfounda-

tions of  $\Omega(\cdot)$  and quantified by the elasticities  $e_s$ ,  $e_v$ , and  $e_t$  where  $e_j = \partial \log \Omega(\cdot) / \partial \log j$ ,  $j = s, v, t$ . Secondly, a wage subsidy leads to a *ceteris paribus* cheapening of labour in the urban sector whereas the income tax makes it less attractive for the migrant. Both these effects make themselves felt only on the equation (2.8) leading to

$$[\Delta'] \begin{bmatrix} \hat{R} \\ \hat{w}_r \\ \hat{\lambda} \end{bmatrix} = \begin{bmatrix} \hat{p}_u - \theta'_{uL} e_T \hat{T} - \theta'_{uL} (1+e_s) \hat{s}_u - \theta'_{uL} (e_v + (\nu_u/1-\nu_u)) \hat{v}_u - \theta'_{uL} e_t \hat{t}_u \\ \hat{p}_r - \theta'_{rL} \hat{s}_r - \theta'_{rL} \nu_r \hat{v}_r \\ e_T \hat{T} - \hat{\tau} + e_s \hat{s}_u + e_v \hat{v}_u + (1+e_t) \hat{t}_u \end{bmatrix} \quad (4.1)$$

where  $\nu_i = v_i/(w_i s_i + v_i)$ ,  $\theta'_{iL} = \theta_{iL} (1-\nu_i)$  and  $\Delta'$  is  $\Delta$  with  $\theta'_{uL}$ ,  $\theta'_{rL}$  substituted for  $\theta_{uL}$ ,  $\theta_{rL}$ .

The following observations now follow from inspection.

##### Proposition 4.1

The effect of rural wage subsidies on urban output and employment is equivalent to a change in rural prices.

##### Proposition 4.2

The effect of urban income taxes on urban output and employment is equivalent to

- (i) a change in urban prices if  $e_t$  equals  $-1$ ; and
- (ii) a change in the distortion for all other values of  $e_t$ .

##### Proposition 4.3

The effect of urban wage subsidies on urban output and employment is equivalent to

- (i) a change in urban prices provided  $e_s$  or  $e_v$  are zero;
- (ii) a change in the differential provided  $e_s$  equals  $-1$  and  $e_v = -(\nu_u/1-\nu_u)$ ; and
- (iii) a change in the distortion for all other values of the elasticities.

Proposition 4.1 coupled with Section 3.3, gives the unambiguous result that rural wage subsidies lead to perverse responses of urban output and employment if and only if  $|\Delta|$  and  $|D|$  conflict. Rural wage subsidies have not been generally discussed in the literature, the Corden-Findlay study being the exception. As pointed out in the introduction,  $|\Delta|$  is always positive in their model and we thus obtain a paradox if and only if the rural sector is more capital-intensive in unemployment-adjusted terms than the rural sector, a fact not explicitly noted by them.



Propositions 4.2 and 4.3 hinge on the values of the elasticities  $e_t$ ,  $e_s$  and  $e_v$  which in turn depend on the labour market conditions emphasized in the model. In the remainder of this section we turn to the examination of this dependence. Our discussion also brings out the innocuous nature of the Standing Hypothesis.

#### 4.1. The Rigid Wage Hypothesis

This is the hypothesis studied by Corden and Findlay [8] and Stiglitz [21]. Here

$$w_u = t_u (\Upsilon s_u + v_u) \quad (4.1.1)$$

where  $\Upsilon$  is the exogenously given rigid wage. Equation (4.1.1) leads to  $e_\lambda = e_w = e_R = 0$ ,  $e_t = 1$ , and positive values of  $e_\Upsilon$ ,  $e_s$  and  $e_v$ . Thus the effects of urban income taxes and wage subsidies are equivalent to changes in the distortion and hence subject to the conclusions obtained in Section 3.5 and collected in Table 2. The ambiguous entries in Table 2 pertaining to changes in  $\Upsilon$  can be made determinate if we make assumptions about the degree of technical substitution as suggested by Corden and Findlay and given precisely in Khan [15].

Finally, note that under this hypothesis, equation (1.4) is always positive.<sup>9</sup>

#### 4.2. The Efficiency Wage Hypothesis

This hypothesis has been systematically studied in the context of immobile capital by Stiglitz [20; 21] whose papers should be seen for further references. It leads to

$$t_u (w_u s_u + v_u) = \Upsilon \quad (4.2.1)$$

where  $\Upsilon$  is the efficiency wage, a constant determined from considerations grounded in nutrition and biological efficiency. Like the rigid wage model, it also gives zero values to  $e_w$ ,  $e_\lambda$  and  $e_R$  and a positive value to  $e_\Upsilon$ ; however, here  $e_s = -1$ ,  $e_t = -\frac{1}{1-\nu_u}$  and  $e_v = \frac{\nu_u}{1-\nu_u}$ . If  $v_u = 0$ , the effect of urban income taxes is equivalent to that of an urban price change; otherwise it is equivalent to changes in the distortion. The effect of a change in the urban wage subsidies is equivalent to changes in the differential. The ambiguous entries pertaining to changes in  $\tau$  can be made determinate if and only if

$$\theta_{rL} - \theta_{rK} \theta_{uL} \quad (4.2.2)$$

<sup>9</sup>This is the case analyzed by Corden and Findlay.

has a determinate sign. Equation (4.2.2) does not involve conditions on  $\sigma_r$  and  $\sigma_u$  as is required for determining the sign of changes in  $\Upsilon$  under both the rigid-wage and efficiency-wage hypotheses.

#### 4.3. The Labour-Turnover Hypothesis

This hypothesis has been systematically studied in the context of immobile capital by Stiglitz [19; 21]. In this setting the urban wage is set by the employer so as to minimize indirect and direct labour costs given by

$$(w_u s_u - v_u) - \Upsilon q(t_u w_u / w_r, \lambda) \quad (4.3.1)$$

where  $q$  denotes quits and is a negative function of the urban-rural wage rate and the unemployment rate and  $\Upsilon$  is a training cost parameter. Minimization of equation (4.3.1) with respect to  $w_u$  leads to the following implicit equation for  $w_u$

$$s_u - \Upsilon t_u (1/w_r) q_1(\cdot) = 0 \quad (4.3.2)$$

By making use of the equilibrium condition, equation (2.3), we obtain

$$w_u = \Upsilon (\tau / s_u) q_1(\tau(1+\lambda), \lambda) \quad (4.3.3)$$

This form of the  $\Omega(\cdot)$  function yields  $e_w = e_R = e_t = e_v = 0$  and  $e_s = 1$ . Stiglitz has shown that under reasonable assumptions on  $q(\cdot)$ ,  $e_\lambda < 0$  [19, equation 18].

Thus, changes in specific wage subsidies are equivalent to urban price changes and changes in urban income taxes and *ad valorem* wage subsidies are equivalent to differential changes. The ambiguous entries pertaining to differential changes in Table 2 is determinate if and only if

$$(e_\lambda (\xi_r \zeta + \kappa_r \xi) - \beta) \text{ and } (e_\lambda \xi_r \phi - \beta) \quad (4.2.3)$$

have determinate signs, where  $\beta = \kappa_r \xi_u (1+\lambda) (\theta_{rL} - \theta_{rK} \theta_{uL})$ .

Stiglitz has discussed all these changes in the immobile capital setting; we refer the reader to his papers for a comparison of his results with ours.

#### 4.4. The Trade Union Hypotheses

In Calvo [6], the endogenous urban wage is determined as a consequence of trade union actions. He assumes capital immobility and investigates two behavioural hypotheses. Under the first, the trade union determines the urban wage so as to maximize a utility function given by  $L_u(w_u - w_r)$  and with the urban employer passively agreeing to the union's actions. Calvo also assumes for the urban sector a



Cobb-Douglas production function with exponent  $\alpha$  and this leads him to his equation (13), given in our notation by

$$w_u = (w_r/\alpha) - v_u(1-\alpha)/s_u\alpha \quad (4.4.1)$$

Under the second behavioural hypothesis, Calvo introduces a Nash arbitrator between the urban employer and the trade union and obtains his equation (29), given in our notation by

$$w_u = ((1+\alpha)/2\alpha) w_r - ((1-\alpha)/2\alpha) v_u/s_u \quad (4.4.2)$$

If we continue to assume a Cobb-Douglas technology and that the union and the arbitrator ignore income taxes in their calculations, equations (4.4.1) and (4.4.2) apply without change to the mobile capital setting. We thus obtain, in either case,  $e_\lambda = e_R = e_t = 0$ ,  $e_w > 0$ ,  $e_s \leq 0$  and  $e_v > 0$ . These values lead to the conclusion that changes in urban taxes and wage subsidies are identical to changes in the distortion. The ambiguous entries pertaining to such changes in Table 2 can be made determinate if and only if equations (3.5.2) and (3.5.3) have determine signs. Finally, note that  $v_u = 0$  implies that  $e_w = 1$  and  $e_s = 0$ . This leads to the result that income tax and *ad valorem* subsidy changes are equivalent to urban price changes.

A more general treatment of both hypotheses can be given but that will be somewhat of a digression from our main theme; in any case, Calvo has discussed the effects of wage subsidies in the immobile capital setting and the use of assumptions identical to his are of advantage for the purpose of comparison.

#### 4.5. The Costly Supervision Hypothesis

This hypothesis has been systematically studied by Calvo and Wellisz and the reader is referred to the overview provided by Calvo [5] for details and references. The discussion is confined almost solely to developed economies but Calvo is obviously aware of the relevance of the ideas to less developed economies. The basic idea is that an employer makes up for lack of supervision by paying a wage higher than what the worker could earn elsewhere. Thus, in our notation, effort  $x$  in the urban sector is given by

$$x = f(t_u w_u - w_r), f(0) \geq 0, f' > 0, f'' < 0 \quad (4.5.1)$$

The urban wage is set so as to maximize urban profits given by

$$p_u F_u(xL_u, K_u) - (w_u s_u + v_u)L_u - RK_u \quad (4.5.2)$$

leading to the following necessary condition:

$$w_u = (s_u w_r + \mu v_u t_u)/(1-\mu)t_u s_u \quad (4.5.3)$$

where  $\mu$  is the elasticity of the effort function given as equation (4.5.1). This form of the  $\Omega(\cdot)$  function yields  $e_\lambda = e_R = 0$  and  $e_w > 0$ . Let us now assume for simplicity that  $\mu$  is constant. If  $v_u = 0$ , the effects of urban income taxes and of *ad valorem* wage subsidies is identical to those generated by an urban price change. If  $v_u \neq 0$ , the effects of changes in these parameters are identical to changes in the distortion. The ambiguous entries in Table 2 pertaining to such changes can be made determinate if and only if equations (4.5.2) and (4.5.3) have determinate signs.

Table 3 summarizes the discussion of this section.

Table 3

*The Equivalence of Various Parameter Changes<sup>1</sup>*

		Price Response	Differential Response	Distortion Response
Rural Subsidies	RW	✓		
	EW	✓		
	LT	✓		
	TU	✓		
	CS	✓		
Urban Income Tax	RW			✓
	EW	(✓) <sup>2</sup>		✓
	LT		✓	
	TU			✓
	CS	(✓) <sup>2</sup>		✓
<i>Ad Valorem</i> Wage Subsidy	RW			
	EW		✓	
	LT		✓	
	TU	(✓) <sup>2</sup>		✓
	CS	(✓) <sup>2</sup>		✓
Specific Wage Subsidy	RW			✓
	EW		✓	
	LT	✓		
	TU			✓
	CS			✓

<sup>1</sup>RW, EW, LT, TU and CS are obvious abbreviations of the five hypotheses discussed in Section 4.

<sup>2</sup>This is in the special case that the specific wage subsidy is zero.



### 5. AN ADJUSTMENT PROCESS

In this section, we turn to question (b) posed in the introduction and present an adjustment process under which an equilibrium is locally asymptotically stable if and only if  $|D|$  and  $|\Delta|$  have identical signs.

Denote by  $P$  an adjustment process which is defined by the following differential equations:

$$\begin{aligned} DK_r &= \phi \left\{ (R_r/R_u) - 1 \right\} & \phi' > 0, \phi(0) &= 0 \\ DL_r &= \psi \left\{ w_r(1+\lambda)/w_u - 1 \right\} & \psi' > 0, \psi(0) &= 0 \\ D\lambda &= \pi \left\{ (\Omega(\cdot)/w_u) - 1 \right\} & \pi' > 0, \pi(0) &= 0 \end{aligned}$$

where  $D$  is the time derivative operator and  $R_i$  is the rental in sector  $i$ ,  $i = u, r$ .  $P$  is very much in the same spirit as the adjustment process presented by Neary [17]. The first two equations show that capital flows into the sector with a higher rental and that labour migrates into a sector with a higher expected wage. The last equation needs somewhat more justification. Recall that the urban wage is being determined by agents in the urban sector in accordance with  $\Omega(w_r, \lambda, R_u, \Upsilon)$ . As the dynamic process unfolds, this calculated urban wage is compared with the one actually prevailing: if it is identical to it, the urban employment decisions remain unchanged; if not, urban labour is hired or fired leading to changes in  $\lambda$ .

In the sequel, we shall take an *equilibrium* to be values of the endogenous variables when  $DK_r = DL_r = D\lambda = 0$ . We now have

#### Assumption 5.1

In equilibrium  $|D| \neq 0$ ,  $|\Delta| \neq 0$  and  $e_w(\sigma_u - 1) \geq 0$ ,  $e_\lambda \leq 0$ ,  $e_R \geq 0$ .

#### Theorem 5.1

Given Assumption 5.1, an equilibrium is locally, asymptotically stable if and only if the unemployment-adjusted factor intensities agree with the elasticities-adjusted factor intensities in their ranking of the two sectors.<sup>10</sup>

Recall that these factor intensities are given by equations (1.3) and (1.4) and denoted by  $|D|$  and  $|\Delta|$ . We now turn to

**5.1 Proof of Theorem 5.1:** In what follows, we shall assume that  $\phi'(0) = \psi'(0) = \pi'(0) = 1$ ; it can be easily checked that this is done without any loss of generality. Linearization of the differential equations around the equilibrium values (denoted by starred superscripts) gives

<sup>10</sup>The part of Assumption 5.1 pertaining to  $e_w$  is not required to show that stability implies  $(|D| \cdot |\Delta|) > 0$ . This is clear from the proof given below.

$$\begin{bmatrix} DK_r \\ DL_r \\ D\lambda \end{bmatrix} = \begin{bmatrix} -\left(\frac{\theta_{rL}}{\sigma_r} + \frac{K_r^*}{K_u^*} \frac{\theta_{uL}}{\sigma_u}\right) & \left(\frac{\theta_{rL}}{\sigma_r} + \frac{L_r^*}{L_u^*(1+\lambda^*)} \frac{\theta_{uL}}{\sigma_u}\right) & \frac{\theta_{uL}}{\sigma_u} \\ \left(\frac{\theta_{rK}}{\sigma_r} + \frac{K_r^*}{K_u^*} \frac{\theta_{uK}}{\sigma_u}\right) & -\left(\frac{\theta_{rK}}{\sigma_r} + \frac{L_r^*}{L_u^*(1+\lambda^*)} \frac{\theta_{uK}}{\sigma_u}\right) & 1 - \frac{\theta_{uK}}{\sigma_u} \\ \left[\frac{e_w \theta_{rK}}{\sigma_r} + \frac{K_r^*}{K_u^*} \frac{(\theta_{uK} + e_R \theta_{uL})}{\sigma_u}\right] & \left[\frac{e_w \theta_{rK}}{\sigma_r} + \frac{L_r^*}{L_u^*(1+\lambda^*)} \frac{(\theta_{uK} + e_R \theta_{uL})}{\sigma_u}\right] a_{33} & \lambda - \lambda^* \end{bmatrix} \begin{bmatrix} K_r - K_r^* \\ L_r - L_r^* \\ \lambda - \lambda^* \end{bmatrix} \quad (5.1.1)$$

where  $a_{33} = e_\lambda - (1/\sigma_u)(\theta_{uK} + e_R \theta_{uL})$ .

We begin by showing that  $(|D| \cdot |\Delta|) > 0$  is sufficient for local asymptotic stability. By a Theorem in Hirsch and Smale [12, p. 181], a sufficient condition for local asymptotic stability of equilibrium is that the  $3 \times 3$  matrix  $A$  in equation (5.1.1) has all eigenvalues with negative real parts. A sufficient condition for this is given by<sup>11</sup>

$$\det(A) < 0; \text{trace}(A) < 0; m = (-a) \text{trace}(A) + \det(A) > 0 \quad (5.1.2)$$

where  $a$  is the sum of principal minors of  $A$  of order two. In what follows we ignore the stars. It can be checked by routine algebra that

$$\det(A) = \frac{-1}{\sigma_u \sigma_r} \frac{L_r}{K_u} \left( \frac{k_u}{1+\lambda} - k_r \right) [\theta_{rL} \theta_{uK} (1 - e_\lambda) + \theta_{uL} e_R] + \theta_{rK} \theta_{uL} (e_\lambda - e_w) \quad (5.1.3)$$

The negativity of the trace is obvious by inspection. All that remains is the sign of  $m$ . On expanding  $\det(A)$  along the third column, we obtain

$$m = -(a_{11} + a_{22} + a_{33})(A_{11} + A_{22}) - A_{23} - (a_{11} + a_{22})A_{33} + (a_{13}A_{13} + \frac{\theta_{uK}}{\sigma_u}A_{23}) \quad (5.1.4)$$

By routine calculations and through the use of Assumption 5.1, this can be shown to be positive.

To show that stability implies  $(|D| \cdot |\Delta|) > 0$ , we have to appeal to another theorem in Hirsch and Smale [12, p. 187]. This assures us that, under local, asymptotic stability, none of the roots of the characteristic cubic of  $A$  can have positive real parts. Since the product of these roots equals  $\det(A)$  which is assumed

<sup>11</sup>This derivation is along the lines found in Neary [17]; see, in particular, the Appendix to his paper. With  $\lambda = 0$ , the first principal minor of order 2 of our matrix is precisely the matrix considered by Neary.



non-zero by Assumption 5.1 (as a consequence of equation (5.1.3)), the only way a root can have a zero real part is by being purely imaginary. Suppose this to be the case, i.e. the three roots are  $\alpha, \pm \beta_1$ , with  $\alpha$  negative and  $\beta$  being of arbitrary sign. But this guarantees that  $\det(A) = -\alpha(-\beta^2) < 0$  and we are home by virtue of equation (5.1.3). Thus, we only need to consider the case when all the roots have negative real parts. But a necessary condition for this is also given by<sup>12</sup> equation (5.1.2) and the proof is again finished by virtue of equation (5.1.3). Q.E.D.

This section will be left incomplete if we do not ask how robust Theorem 5.1 is with respect to changes in the underlying adjustment process. Neary poses this question in the context of his wage-differential setting and concludes, "while it is not difficult to devise alternative mechanisms, there is no reason to believe that they imply different stability conditions." Unfortunately, this is no longer true in the rich setting of the generalized Harris-Todaro model. Indeed  $(|D| \times |\Delta|) > 0$  is *not* equivalent to stability of equilibrium of a rather obvious adjustment process of a Walrasian kind; for example

$$Dw_r = L_r + L_u (w_u/w_r) - \mathcal{L}$$

$$DR = K_r + K_u - \kappa$$

$$Dw_u = \Omega(w_r, w_u/w_r, R) - w_u.$$

To see this, all one needs is to evaluate the determinant<sup>13</sup> of the matrix underlying the linearized differential equations.

It is not clear to me how much significance should be attached to this lack of robustness. What is nice about Neary's paper and the corresponding results of this paper is that there do exist intuitively reasonable adjustment processes under which a correspondence between comparative-statics results and stability can be had. The paucity of comparative-statics results in general equilibrium theory,<sup>14</sup> even with hypotheses guaranteeing stability, has taught us a while ago that one must take what

<sup>12</sup>See, for example, Coppel [7, p. 158] and Ayres [2, Problem 1, p. 152].

<sup>13</sup>The relevant matrix is

$$\begin{bmatrix} -\frac{1}{w_r} \left( \frac{L_r \sigma_r}{\theta_{rK}} + (1+\lambda)L_u \right) & \frac{1}{R} \left( \frac{L_r \sigma_r}{\theta_{rL}} + \frac{(1+\lambda)L_u \sigma_u}{\theta_{uL}} \right) & \frac{L_u}{w_r} \left( 1 - \frac{\sigma_u}{\theta_{uK}} \right) \\ \frac{K_r \sigma_r}{w_r \theta_{rK}} & -\frac{1}{R} \left( \frac{K_r \sigma_r}{\theta_{rL}} + \frac{K_u \sigma_u}{\theta_{uL}} \right) & \frac{K_u \sigma_u}{w_u \theta_{uK}} \\ (1+\lambda)(e_w - e_\lambda) & (w_u e_R / R) & (e_\lambda - 1) \end{bmatrix}$$

<sup>14</sup>See especially the conclusion to Chapter 10 in Arrow and Hahn [1].

one can get. Secondly, the theory of tâtonnement adjustment processes is sufficiently crude<sup>15</sup> in its economics so as to make a search for "realistic" adjustment processes somewhat misplaced. Nevertheless, this lack of robustness of Theorem 5.1 is of some relevance when the object is, in part, to show that the coherence brought out by Neary does not fully extend to a richer and more complicated theory of factor market distortions.

## 6. IMPLICATIONS FOR COMMERCIAL POLICY

This section brings out the implications of the results in Sections 3 to 5 for welfare theory. We begin with an analysis of a "large" economy, i.e. the one with monopoly power in trade. Later on we shall specialize to an economy which takes international prices as given.

Following Srinivasan and Bhagwati [18], a "large" economy is formalized by letting  $E$  units of the agricultural good exchange for  $h(E)$  units of the manufacturing good where

$$h(0) = 0, h' > 0, h'' < 0 \quad (6.1)$$

A positive  $E$  is interpreted as net exports and equation (6.1) brings out the fact that the marginal and average terms of trade decline as  $E$  increases.

Welfare is measured by a concave utility function  $U(\cdot)$  which is defined on domestic consumption  $(Z_r, Z_u)$  and which has positive marginal utilities for each good. Feasibility requires

$$Z_r = X_r - E \text{ and } Z_u = X_u + h(E) \quad (6.2)$$

where  $X_r$  and  $X_u$  are the supplies of the two commodities, properties of which have been discussed at length in the sections above.

In the Srinivasan-Bhagwati setting, capital is immobile and the urban wage is an exogenously given constant. These production conditions coupled with equations (6.1) and (6.2) allow them to characterize the equilibrium and investigate how the value of social welfare changes as this equilibrium is disturbed by a policy parameter. They carry out this investigation in terms of the primal variables, i.e. with the marginal rates of substitution substituted for the prices. This involves considerable algebra and their approach becomes especially complicated in our setting with mobile capital and with the urban wage given by  $\Omega(\cdot)$ . We work with more natural independent variables in the dual setting.

Let  $g(p_r, p_u, U)$  be the minimum expenditure with given prices  $p_r, p_u$  required to reach a level of welfare  $U$ . It is well known (see, for example, Gorman [9]) that  $g$

<sup>15</sup>See especially the conclusion to Chapter 12 in Arrow and Hahn [1].



is (i) positively homogeneous of degree one in prices, (ii) a concave function of the prices, and (iii) that  $g_i = \partial g / \partial p_i = Z_i$  ( $i = u, r$ ). In addition, the assumption of both goods being normal implies  $g_{io} = \partial^2 g / \partial p_i \partial U$ ,  $i = u, r$ , are positive. We can now write the balance of trade and feasibility equations as

$$E - ph(E) = 0; p = p_u/p_r \quad (6.3a)$$

$$g_r - X_r = -E \text{ and } g_u - X_u = h(E) \quad (6.3b)$$

with  $p$ ,  $E$  and  $U$  as the unknowns. The effect of a change in welfare for a change in a parameter  $a$  is simply  $dU/da$  which can be extracted from

$$\begin{bmatrix} -h(E) & 1 - ph'(E) & 0 \\ g_{ru} - (\partial X_r / \partial p) & 1 & g_{ro} \\ g_{uu} - (\partial X_u / \partial p) & -h'(E) & g_{uo} \end{bmatrix} \begin{bmatrix} dp \\ dE \\ dU \end{bmatrix} = \begin{bmatrix} 0 \\ (\partial X_r / \partial a) da \\ (\partial X_u / \partial a) da \end{bmatrix} \quad (6.4)$$

It is easy to show using equation (6.1), along with the elementary properties of the expenditure function listed above, that the determinant,  $|B|$ , of the matrix in equation (6.4) is negative if the supply responses are well-behaved, i.e.  $\partial X_r / \partial p < 0$  and  $\partial X_u / \partial p > 0$ . Denote by  $C$  the matrix obtained by substituting  $\partial X_i / \partial a$  for the entries  $g_{io}$ , ( $i = u, r$ ), in the matrix  $B$ . Application of Cramer's rule then gives

$$(dU/da) = |C|/|B| \quad (6.5)$$

Since  $g_{io}$ ,  $i = u, r$ , are both positive and  $(\partial X_i / \partial a)$ ,  $i = u, r$ , are generally of opposite signs,  $|C|$  will, in general, be unsigned. This is just another way of saying that welfare can increase or decrease as a consequence of changes in  $a$ . Thus equation (6.5) can be used to characterize the optimal values of  $a$ , i.e. the equation  $|C|/|B| = 0$ . However, rather than give necessary conditions for the optimality of a second-best policy, as in the various appendices in Stiglitz [21] or in Khan [15], Bhagwati and Srinivasan [3; 18] look for sufficient conditions under which a small perturbation of a particular parameter of a distorted economy improves welfare, i.e. sufficient conditions for  $dU/da$  to be positive. Their analysis implicitly relies on the observation that use of equations (2.2), (2.3) and (2.6) yields<sup>16</sup>

$$p \frac{\partial X_u}{\partial a} = - \left( \frac{w_r s_u}{t_u} + \frac{v_u}{(1+\lambda)} \right) \left( \frac{\partial L_r}{\partial a} + L_u \frac{\partial \lambda}{\partial a} \right) - R \frac{\partial K_r}{\partial a} \quad (6.6)$$

<sup>16</sup>Both (2.2) and (2.3) have to be modified to incorporate wage subsidies and income taxes, i.e.,  $s_u$ ,  $v_u$  and  $t_u$ .

and if the initial equilibrium is one of *laissez faire*, i.e.  $s_u = t_u = 1$  and  $v_u = 0$ , one obtains the further simplification that

$$p(\partial X_u / \partial a) = -(\partial X_r / \partial a) - w_r L_u (\partial \lambda / \partial a). \quad (6.7)$$

This allows us to rewrite equation (6.5) as

$$p(|B|) \frac{dU}{da} = \left( \frac{\partial X_r}{\partial a} \right) \left\{ (1 - ph') (h + g_{ru} + pg_{uu} - \frac{\partial X_r}{\partial p} - p \frac{\partial X_u}{\partial p}) \right\} + w_r L_u \left( \frac{\partial \lambda}{\partial a} \right) \left\{ h + (1 - ph') (g_{ru} - \frac{\partial X_r}{\partial p}) \right\} \quad (6.8)$$

Under some further simplifications<sup>17</sup> and based on the discussion in Section 3, we can now provide sufficient conditions under which small perturbations of various policy instruments lead to improvements in welfare starting from initial positions of *laissez faire*. We leave it to the interested reader to provide such a catalogue. What should be emphasized, however, is that stability of our adjustment process is not sufficient by itself. To see this, go back to equation (6.8) and consider a situation when a change in  $a$  implies a change in the distortion. Here  $(|D| \times |\Delta|) > 0$  is sufficient to sign  $\partial \lambda / \partial a$  but *not*  $\partial X_r / \partial a$ . For situations when a change in  $a$  implies a price change, the situation is reversed and for the case of a differential change,  $(|D| \times |\Delta|) > 0$  is not sufficient to sign either the output or the unemployment rate changes. However, the flavour of the results changes considerably when we shift from a "large" economy to a "small" one.<sup>18</sup>

In the context of a "small" economy in which  $p_r$  and  $p_u$  are taken as exogenous parameters, equation (6.3) collapses into

$$g(p_r, p_u, u) = p_r X_r + p_u X_u. \quad (6.9)$$

Routine differentiation of this, along with the rewriting of the third row of equation (2.9) in terms of  $|D|$  and  $|\Delta|$ , gives our next result.

### Theorem 6.1

Let the economy have no monopoly power in trade. Then given an initial *laissez faire* equilibrium, there exist welfare-improving changes in

- (i) the distortion if and only if  $(|D| \times |\Delta|) > 0$
- (ii) the differential if and only if  $-e_\lambda (|D|/|\Delta|) > 1$

<sup>17</sup>Note that the first term on the right hand side of (6.8) can be simplified further. Given positive homogeneity of degree 0 of  $g_u$ ,  $g_{ur} + pg_{uu}$  equals zero. Similarly an analog of (6.7) for changes in  $p$  yields  $-(\partial X_r / \partial p + p \partial X_u / \partial p)$  equal to  $w_r L_u \partial \lambda / \partial p$ .

<sup>18</sup>This is not so for the Bhagwati-Srinivasan setting. Compare Theorem 2, for example, of Bhagwati and Srinivasan [3] with the corresponding one in Srinivasan and Bhagwati [18].



- (iii) the urban price if and only if  $(1-e_\lambda)(|D|/|\Delta|) > (Z_u/X_u)$
- (iv) the rural price if and only if  $(e_w-e_\lambda)(|D|/|\Delta|) > \frac{\mathcal{L}-L_r(Z_r/X_r)}{\mathcal{L}-L_r}$

Our primary interest, of course, is in the existence of welfare-improving changes in wage subsidies and income taxes; but then Propositions 4.1 to 4.3 and Table 3 relate changes in these policies to the abstract changes discussed in Theorem 6.1. This result is thus a far-reaching generalization of Theorem 2 in Bhagwati and Srinivasan [3]; it allows capital mobility, a variety of subsidies and taxes, and a variety of labour market conditions in the urban sector. It is also a rather nice result in that it brings out how far one can go in characterizing welfare-improving changes from initial positions of *laissez faire* just by the knowledge that the unemployment-adjusted and the elasticities-adjusted factor intensities give identical rankings of the two sectors.

## 7. CONCLUDING REMARKS

We have shown that the Corden-Findlay paradox and, more generally, perverse responses of urban output and employment to wage subsidies do not correspond to instability of equilibrium for a suitably formulated adjustment process. However, this lack of correspondence is by no means universal and depends on the labour-market conditions prevailing in the urban sector. We have also shown that the Bhagwati-Srinivasan proposition is valid, for a small economy, under conditions which correspond to stability of equilibrium for the aforementioned adjustment process. However, this too is not universal and depends on urban labour-market conditions. We were also able to show that these conditions are not sufficient by themselves for the validity of the Bhagwati-Srinivasan proposition for a "large" economy.

Throughout the analysis presented above, we have assumed that there exists a locally unique equilibrium which is stable under small perturbations. These issues are discussed at some length in Khan [15] and to go into them in any detail here would carry us too far afield.

We have ignored taxes and subsidies on capital and confined ourselves solely to a small open economy. The reader can work out for himself the consequences of such taxes. Our results could also be extended to a closed economy in the spirit of Jones [14] and Neary [17]; it is doubtful whether this extension would add anything to our basic point that there is considerably unity to the Harris-Todaro literature and that this unity could be grasped more easily if we keep the simple differential wage setting as the relevant backdrop.

Finally, it must be pointed out, as Corden and Findlay and others have done before, that there is no government budget restraint. One can assume that all subsidies are being financed by a tax on capital and this makes no difference to the analysis given that it is inelastically supplied. However, with capital accumulation we have a different story; but to model that, one has to turn to growth theory.

## REFERENCES

1. Arrow, K. J., and F. H. Hahn. *General Competitive Analysis*. San Francisco: Holden Day, Inc. 1971.
2. Ayres, F. *Matrices*. New York: Schaum Publishing Company. 1962.
3. Bhagwati, J., and T. N. Srinivasan. "The Ranking of Policy Interventions Under Factor Market Imperfections: The Case of Sector-Specific Sticky Wages and Unemployment". *Sankhya*, Series B. 1973.
4. Bhagwati, J., and T. N. Srinivasan. "The Theory of Wage Differentials: Production Response and Factor Price Equalization". *Journal of International Economics*. Vol. 1. 1971. pp. 19-35.
5. Calvo, Guillermo A. "Quasi-Walrasian Theories of Unemployment". *American Economic Review*. Vol. 69. 1979. pp. 102-107.
6. Calvo, Guillermo A. "Urban Unemployment and Wage Determination in LDC's: Trade Unions in the Harris-Todaro Model". *International Economic Review*. Vol. 19. 1978. pp. 65-81.
7. Coppel, W. A. *Stability and Asymptotic Behavior of Differential Equations*. Boston: D. C. Heath and Company. 1965.
8. Corden, W. M., and R. Findlay. "Urban Unemployment, Intersectoral Capital Mobility and Development Policy". *Economica*. Vol. XLII. 1975. pp. 59-78.
9. Gorman, W. M. "Tricks with Utility Functions". In M. Artis and A. Nobay (eds.), *Essays in Economic Analysis*. Cambridge: Cambridge University Press. 1976.
10. Harris, J. R., and M. Todaro. "Migration, Unemployment and Development: A Two Sector Analysis". *American Economic Review*. Vol. LX. 1970. pp. 126-142.
11. Hazari, B. R. *The Pure Theory of International Trade and Distortions*. New York: John Wiley and Sons. 1978.
12. Hirsch, M. W., and S. Smale. *Differential Equations, Dynamical Systems and Linear Algebra*. New York: Academic Press. 1974.
13. Jones, R. W. "The Structure of Simple General Equilibrium Models". *Journal of Political Economy*. Vol. 73. 1965. pp. 557-572.
14. Jones, R. W. "Distortions in Factor Markets and the General Equilibrium Model of Production". *Journal of Political Economy*. Vol. 79. 1971. pp. 437-459.



15. Khan, M. Ali. "The Harris Todaro Hypothesis and the Heckscher-Ohlin-Samuelson Trade Model: A Synthesis". *Johns Hopkins Working Paper*, No. 51. 1979. (Forthcoming in *Journal of International Economics*).
16. Magee, S. P. *International Trade and Distortions in Factor Markets*. New York and Basel: Marcel-Dekker, Inc. 1976.
17. Neary, J. P. "Dynamic Stability and the Theory of Factor-Market Distortions". *American Economic Review*. Vol. 68. 1978. pp. 672-682.
18. Srinivasan, T. N., and J. Bhagwati. "Alternative Policy Rankings in a Large, Open Economy with Sector-Specific, Minimum Wages". *Journal of Economic Theory*. Vol. 11. 1975. pp. 356-371.
19. Stiglitz, J. E. "Alternative Theories of Wage Determination and Unemployment in LDC's: The Labor-Turnover Model". *Quarterly Journal of Economics*. Vol. LXXXVIII. 1974. pp. 194-227.
20. Stiglitz, J. E. "The Efficiency Wage Hypothesis, Surplus Labor, and the Distribution of Income in the LDC's". *Oxford Economic Papers*. Vol. 28. 1976. pp. 185-207.
21. Stiglitz, J. E. "The Structure of Labor Markets and Shadow Prices in LDC's". Forthcoming in R. H. Sabot (ed.), *Essays on Migration and the Labor Market in Developing Countries*, to be published by Oxford University Press.