On the Concept of Foreign Exchange Multiplier—A Correction

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In an earlier issue of this journal, Diamond [1] has argued that in developing countries increased imports may have an inflationary rather than a deflationary impact on the economy. His reasoning is based on the fact that developing countries are faced with short supplies of imported inputs and not with a deficient demand. An increase in the imports of intermediate goods results in increased production and higher G.N.P. The ratio of the increase in output to the increase in the imports is termed foreign exchange multiplier by Diamond.

Diamond's analysis is quite useful as it enables one to determine the increase in output when foreign exchange constraint is relaxed in a developing economy. However, his analysis suffers from two problems. First, the assumption that all of the increase in foreign exchange will be allocated to the imports of intermediate goods is unrealistic. Second, there is a mathematical error in his Equation 6 when he divides the output vector by an imports vector. In this note, the assumption that all of the increase in foreign exchange is allocated to the imports of intermediate inputs is relaxed. Diamond's Equation 6 is corrected. It is further shown that for the computation of a foreign exchange multiplier for the economy as a whole one does not need inter-industry matrix and that information regarding the value added per unit of gross output is sufficient.

I. A Flaw in the Analysis and its Correction

Two-Sector Model

As in Diamond's study, the output X is divided into intermediate demand, aX, and final demand, fX, implying $a+f=1$. An initial increase in the output

of X will set up chain reaction. The total increase in the production of X, due to a initial change in the output of X, is

$$\Delta x + (1-f)\Delta x + (1-f)^2 \Delta x + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad \text{I}$$

which sums up to

$$\frac{\Delta x}{f} = \frac{\Delta x}{1-a}$$

If m units of imports are required to produce one unit of X, then the increase in foreign exchange for the imported inputs (ΔM_r) would be

$$\Delta M_r = \frac{m}{1-a}\Delta x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad \text{II}$$

We assume that all the increase in the availability of foreign exchange will not be allocated to the imports of intermediate goods. If k is the proportion of imports allocated to the imports of raw materials, then

$$\Delta M_r = k\Delta M \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad \text{III}$$

Substituting III in II, we get

$$\frac{\Delta x}{\Delta M} = \frac{k}{m}(1-a) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad \text{IV}$$

Compare IV with Diamond's equation 3 which is

$$\frac{\Delta x}{\Delta M} = \frac{1-a}{m}$$

Since $k \leq 1$, the multipliers reported by Diamond would, in general, overstate the true multiplier effect.

Multisector Model

A multisector model may be specified as

$$(I - A)\Delta x = \Delta s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad \text{V}$$

where

$$A = \text{matrix of technical coefficients,}$$
$$\Delta x = \text{column vector of output level, and}$$
$$\Delta s = \text{column vector of final supply.}$$

Premultiplying V by $(I - A)^{-1}$, we get

$$\Delta x = (I - A)^{-1}\Delta s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad \text{VI}$$

Let M be a diagonal matrix. The diagnoal elements of this matrix are per unit import requirements.

$$\Delta m_r = M\Delta x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad \text{VII}$$

where Δm_r is the required increase in the imported inputs.

From III, VII and VI we get

$$\triangle m = \frac{M}{k} [I-A]^{-1} \triangle s \dots\dots\dots\dots\dots\dots\dots \quad VIII$$

$$\triangle s = k [I-A] M^{-1} \triangle m \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad IX$$

Since both $\triangle s$ and $\triangle m$ are vectors, it is not possible to write $\triangle s / \triangle m$. Our interest lies in the effect of change in imports on an industrial activity (or economy). It is shown in Appendix that

$$\frac{\triangle s_i}{\triangle n} = k \left[1 - a_{ii} \frac{r_i}{M_i} - \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij} \frac{r_j}{M_j} \right] \dots\dots\dots\dots \quad X$$

where $\triangle n = \Sigma \triangle m_i$ and r_i is the share of the ith industry in imported raw materials.

It is further shown in the Appendix that

$$\frac{\triangle c}{\triangle n} = k \sum_i \frac{r_i}{m_i} VA_i \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad XI$$

where $\triangle c$ is the increase in supply in the economy and VA_i is value added in the ith sector. In the case of only one activity, (XI) reduces to (IV).

It can easily be seen from XI that to estimate the foreign exchange multiplier for the economy as a whole, all we need to know are value added in different sectors, the import coefficients, and the distribution of foreign exchange by sectors. Knowledge of inter-industry matrix for the estimation of the multiplier is not necessary. For the estimation of sectoral multipliers, one would still need the inter-industry matrix, however.

II. Concluding Remarks

We can be brief in conclusion. An error is noted in Diamond's mathematical analysis. Our analysis shows that the matrix of inter-industry coefficients is not required for the computation of the foreign exchange multiplier for the entire economy. It is further shown that Diamond's assumption that incremental imports consist exclusively of intermediate commodities over-estimates the foreign exchange multiplier relative to the situation when an allowance is made for non-intermediate commodity imports.

From relation IX, we have

$$\triangle s = k [I-A] M^{-1}.m$$

This relation gives

$$\triangle s_1 = k \left[(1-a_{11}) \frac{\triangle m}{M_1} - a_{12} \frac{\triangle m_2}{M_2} \ldots + a_{1n} \frac{\triangle m_n}{M_n} \right]$$

$$\vdots \qquad \qquad \vdots \qquad \qquad \vdots \qquad \qquad \vdots \tag{I}$$

$$\triangle s_n = k \left[-a_{n1} \frac{\triangle m_1}{M_1} - a_{n2} \frac{\triangle m_2}{M_2} \ldots + (1-a_{nn}) \frac{\triangle m_n}{M_n} \right]$$

Let $r_i = \triangle m_i / \triangle n$ be the share of the ith industry in imported raw materials,

where $\triangle n = \Sigma \triangle m_i$

Using this definition, we get

$$\frac{\triangle S_1}{\triangle n} = k \left[(1-a_{11}) \frac{r_1}{M_1} - a_{12} \frac{r_2}{M_2} \ldots a_{1n} \frac{r_n}{M_n} \right]$$

$$\vdots \qquad \qquad \vdots \qquad \qquad \vdots \qquad \qquad \vdots \tag{II}$$

$$\frac{\triangle S_n}{\triangle n} = k \left[-a_{n1} \frac{r_i}{M_1} - a_{n2} \frac{r_2}{M_2} \ldots + (1-a_{nn}) \frac{r_n}{M_n} \right]$$

i.e $$\frac{\triangle s_i}{\triangle N} = k \left[(1-a_{ii}) \frac{r_i}{M_i} - \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij} \frac{r_j}{M_j} \right] \tag{III}$$

To obtain a multiplier, relating the increase in production in the economy due to increase in availability of imported inputs, one needs only to sum

$\frac{\triangle s_i}{\triangle N}$ over all i, i.e.

$$\frac{\Delta c}{\Delta n} = \frac{1}{\Delta n} \Sigma \Delta s_i = k \left[\frac{r_1}{M_1} (1 - a_{11} \ldots - a_{n1}) + \ldots + \frac{r_n}{M_n} (1 - a_{1n} \ldots a_{nn}) \right] =$$

$$k \left[\frac{r_1}{M_1} VA_1 + \ldots + \frac{rN}{M_n} VA_n \right] \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad \text{(IV)}$$

$$k = \Sigma \frac{r_i}{M_i} VA_i$$

Reference

1.   Diamond, J. "On the Concept of the Foreign Exchange Multiplier." *Pakistan Development Review.* Vol. XIV, No. 2. Summer 1975.