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Population Growth and GDP Growth in Pakistan: Three Models

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Three dynamic population—GDP growth models, are developed, each based on a Cobb-Douglas production function. The first model simulates the effects of average wealth on population and GDP growth, whereas the second model simulates the effects of wealth inequality. The third model simulates the effects of a demographic dividend. The models successfully simulate the observed historical and projected future population and GDP of Pakistan. Scenario simulations with the first model with higher and lower population growth rates result in larger or smaller GDP, respectively, but smaller or larger GDP per capita at 2100. The inequality model simulations with reducing or increasing inequality result in a smaller or larger population, respectively, and smaller or larger GDP, but higher or lower GDP per capita at 2100. The demographic dividend model simulations result in larger GDP and higher GDP per capita than the other models.

JEL Classification: C60, J10, O10
Keywords: Endogenous Growth, Dynamic Model, Demographic Transition, Inequality

1. INTRODUCTION

Pakistan's vision is to become a high-income country by 2047 (Planning Commission, 2014). However, rapid population growth and modest economic growth present many challenges in achieving this vision (Planning Commission, 2014, World Bank, 2019). Population growth and economic growth are linked, and reducing the rate of population growth is central to boosting economic growth (Planning Commission, 2014; World Bank, 2019). Reducing population growth and boosting economic growth would also help Pakistan meet other challenges, such as ensuring water security (Kirby, et al. 2017).

In the decade to 2020, the population of Pakistan grew by a little more than 2 percent per year (UN, 2019a), and its gross domestic product (GDP) grew by about 4 percent per year (PBS, 2020). Projecting future population and economic growth is necessary to plan for future needs across many sectors, such as the water sector noted above. My aim in this paper is to examine three candidates dynamic population—GDP growth models for making long-term projections. Each is based on a Cobb-Douglas type of production function. While these models are based on Pakistan, they are quite general and can be applied to any country or region.

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Note: A great many papers explore the issues described in what follows. Some key papers are given as examples, with emphasis on papers about Pakistan. No attempt is made here to comprehensively review this large literature.

Production function models include neo-classical economics models, the classic example of which is that presented by Solow (1956) who proposed a long-run economic growth model of whole economies, based on a production function with decreasing returns to capital. Population growth and savings were treated as exogenous. Critiques of this and similar models noted that they failed to account for the increasing growth observed over the long run in many economies (Shaw, 1992; Howitt, 2010). The critiques led to endogenous growth models in which economic growth was also a function of technological progress based on knowledge accumulation (Shaw, 1992; Howitt, 2010). There are many such models, with different assumptions and differing predictions (Shaw, 1992; Romer, 1994). In some versions, knowledge accumulation itself is a function of population size (Birchenall, 2016; Bucci & Prettner, 2020). One application of such models has been to attempt to explain the differences in growth rates amongst countries, and suggest policies for enhancing growth (Mankiw, et al. 1992; Shaw, 1992; Romer, 1994). In the case of Pakistan, endogenous growth models have been applied to examine the impact on the economic growth of foreign direct investment (Falki, 2009), of education (Abbas, 2001), and of human capital and total factor productivity (Amjad & Awais, 2016). The latter authors note that in recent years a decline in total factor productivity growth is associated with slower economic growth than experienced in nearby and broadly comparable countries. Saleem, et al. (2019) also noted the weak growth in total factor productivity in Pakistan and its contribution to weak economic growth.

Development of these models is the unified growth theory (Galor, 2005), which describes the joint evolution of education, technological progress, resource use, and population; these variables are used in a production function to give the overall output of the economy. Unified growth theory models use a dynamic, time-stepping approach, in which the values of the variables (for population and so on) are updated iteratively in each time step. A key feature of the models is that fertility rates and mortality rates are functions of household preferences, as influenced by wealth and education. These models have been applied to very long-run development and display two regimes, in the first of which there is limited growth, and in the second of which a demographic transition leads to substantial and sustained economic growth. This is consistent with very long-run human development before and after the industrial revolution (Galor, 2005, 2010; Strulik & Weisdorf, 2008).

A demographic transition, in which birth rates fall and per capita wealth rises, is often cited as a key factor in economic growth in developing economies (Bloom & Williamson, 1998; Cervellati, et al. 2017; Bawazir, 2019). The falling birth rates lead to a period in which the fraction of the population that is of working age increases, and there is thus a decline in the number of dependents per worker. The effect lasts a few decades, after which the ageing population results in many older people to be supported by those of working age (Bloom & Williamson, 1998; Ahmad & Khan, 2019). Bidisha, et al. (2020) suggested that in Asia, the demographic transition has limited short-run effects, but significantly boosts economic growth in the long run; Iqbal, et al. (2015) concluded the same for Pakistan. However, Amjad (2013) found that in Pakistan, the onset of a demographic transition from the 1990s appears not to have stimulated economic growth. Choudry & Elhorst (2010) found that the impact of demographic transition is less in

Pakistan than in India and China, though they expect the impact to be greater in the future. Bloom, et al. (2011) also noted that the demographic transition in Pakistan started somewhat later than some of its neighbours, and caution that (for all countries) capturing the benefits is not automatic but depends on governance, trade, and macro-economic management. Bongaarts, et al. (2013) also noted the importance of education and gender equity, while Jehan & Khan (2020) noted the importance of employment opportunities. Ahmad & Khan (2019) developed a model of economic growth which incorporates age structure and a demographic transition, which they used as a basis for an econometric model of human capital dynamics.

Although the impact is debated (Sinding, 2009), poverty is also often cited as a factor in economic growth. Sinding (2009) regards recent evidence as conclusive; reducing poverty leads to lower birth rates, which in turn contributes to economic development. Tahir, et al. (2014) and Afzal, et al. (2012) found that poverty and GDP growth are associated in Pakistan. Afzal, et al. (2012) found that the causality in Pakistan is bi-directional, whereas others such as Chani, et al. (2011) and Cheema & Sial (2012) assume that poverty (or its alleviation) is a result of economic growth. Nevertheless, the latter authors find a positive association between growth and poverty.

A positive association between economic growth and poverty means that with a larger economy, there is less poverty, even if the relative distribution of income remains unchanged. However, income inequality itself may play a role (e.g. Galor, 2011). Shahbaz, et al. (2014) and Soharwardi, et al. (2018) found that income inequality in Pakistan is positively associated with economic growth; Shahbaz, et al. find that the causality is bi-directional. However, Asad, et al. (2011) found that the picture is more complicated, with periods of declining consumption inequality and periods of increasing inequality. Income inequality may also be linked to population growth. Qureshi & Arif (2001); ADB (2002); Hyder, et al. (2010); Arif (2013); Majeed & Malik (2015); Ibrahim, et al. (2019) all found a positive association between poverty and family size. Baulch and McCulloch (1998) analysed a five-year longitudinal survey and found that larger households increased the probability of entering poverty and decreased the probability of exiting. However, as shown by Naschold (2009) several other factors, in particular education, influence moving out of poverty.

My aim in this paper is to present three candidate dynamic population—GDP growth models for Pakistan, each based on a Cobb-Douglas type of production function. The models arise from the factors noted above about the relationships amongst GDP growth, population, wealth inequality, and demographic dividend. The models are based on the example of the unified growth theory models of Galor (2005, 2010, 2011), but are more restricted in scope. In all three models, GDP and population co-evolve: GDP growth results from an increase in population, and the growth in population is in turn influenced by GDP growth. In the first model, which I term the simple model, GDP is related to the size of the total population. In the second model, the wealth inequality model, GDP is related to the size of the population and the distribution of wealth amongst the population. In the third model, the demographic dividend model, GDP is related to the size of the working population and to the ratio of the non-working population to the working population. I examine the behaviour of the three models from 1960 to 2100. From 1960 to the present, the simulations are compared to historical data, whereas from 2020 onwards they are compared to projections of population and GDP growth.

In developing the models, I do not seek to test explanations of GDP and population growth. I accept the evidence from the literature noted above. As noted above, endogenous growth models have been applied to Pakistan, and the historical associations between GDP growth and population growth are well studied. However, I am not aware of any study that uses a dynamic model of the co-evolution of GDP and population in Pakistan. The novel contribution in this paper is the development of models that satisfactorily simulate this co-evolution.

The paper is organised as follows. In the next section, I will outline the GDP and population growth in Pakistan from 1960, with projections from 2020 to 2100. In Section 3, I present the three models, followed in Section 4 by examining the results of model simulations, and their comparison to historical and projected data. In Section 5, I discuss the results in light of other literature, and also discuss influences on GDP and population growth that are not incorporated into the models. This is followed by some overall conclusions.

2. GDP GROWTH AND POPULATION GROWTH IN PAKISTAN: HISTORICAL TRENDS AND FUTURE PROJECTIONS

Historical GDP data were obtained from Table 3 of the Pakistan national accounts (PBS, 2020) which gives the main aggregates at constant prices from 1960-61 to 2019-20. PWC (2017) project that GDP growth at constant purchasing power parity in Pakistan to 2050 will average 4.3 percent per annum. The growth rate was projected to decline slightly over the period. For a projection from 2020 to 2100, therefore, I assumed a constant GDP growth rate of 4.1 percent per annum, slightly less than the PWC (2017) average value to 2050. Figure 1 shows the growth of GDP over the whole period.

Fig. 1.

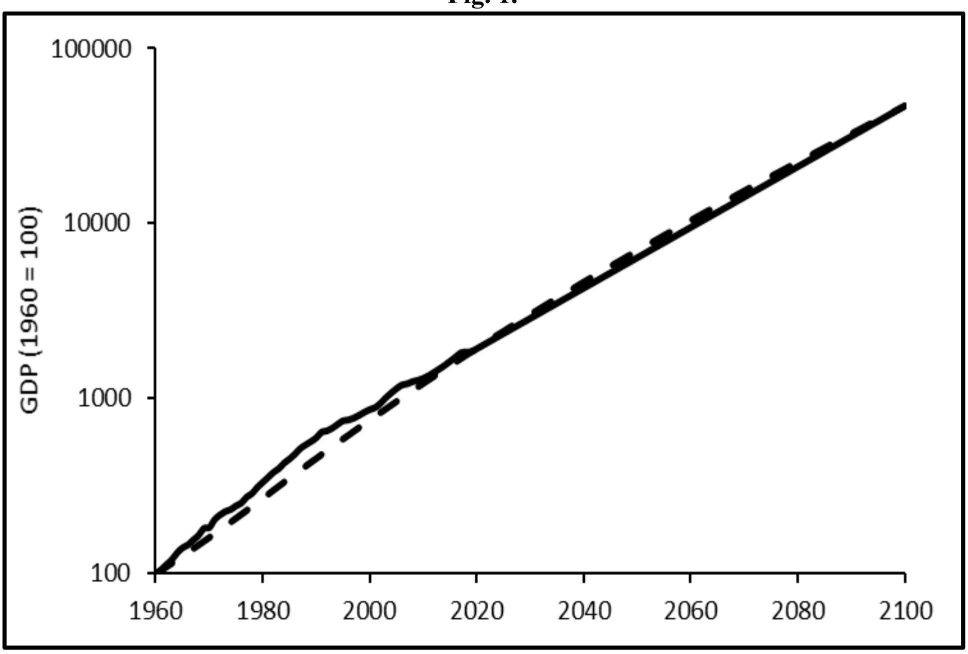
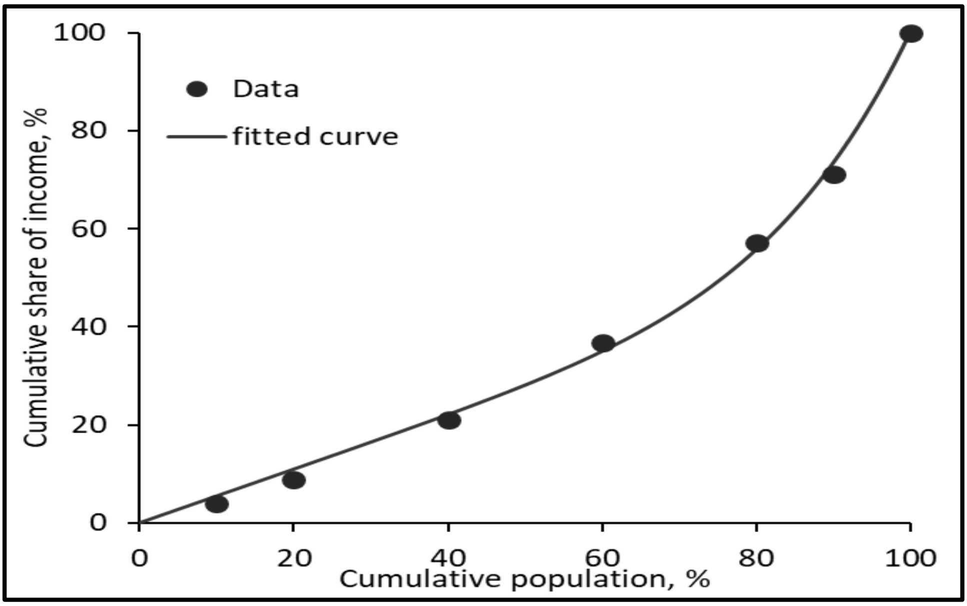


Figure 1. GDP in Pakistan at constant prices plotted as an index with 1960 = 100. The solid line is based on the data from PBS (2020) for 1960-2020, extrapolated at 4.1 percent per year from 2020-2100. The dotted line is the result of a simulation using the simple model (see results section for explanation).

Income distribution data were obtained from the World Bank Poverty and Equity Database (World Bank, 2020). The data give income shares from 1987 to 2015 of the bottom 10 percent and top 10 percent of the population, plus each population quintile. Figure 2a shows the data plotted as a Lorentz curve. The calculated Gini coefficient from 1987 to 2015 is shown in Figure 2b. There is no significant trend in the Gini coefficient.



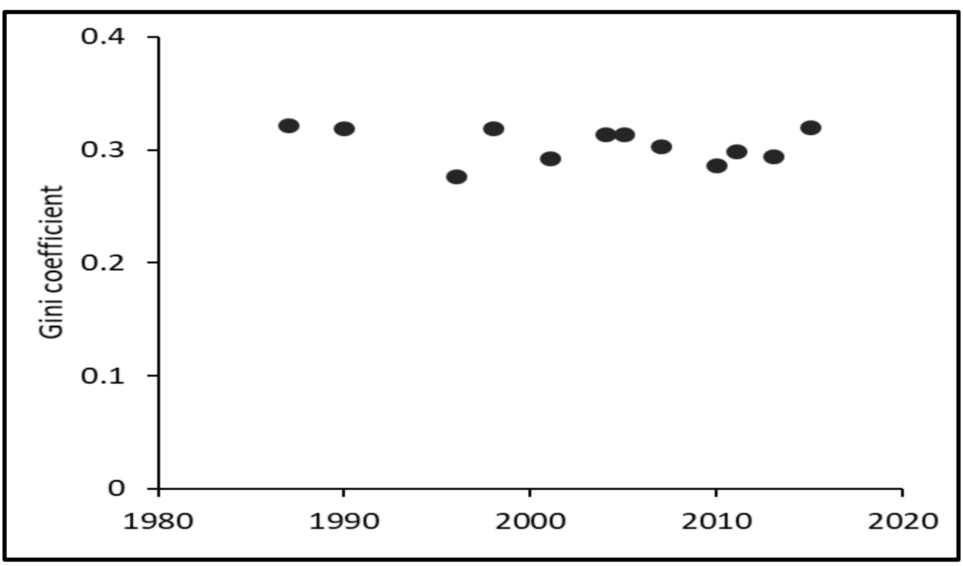
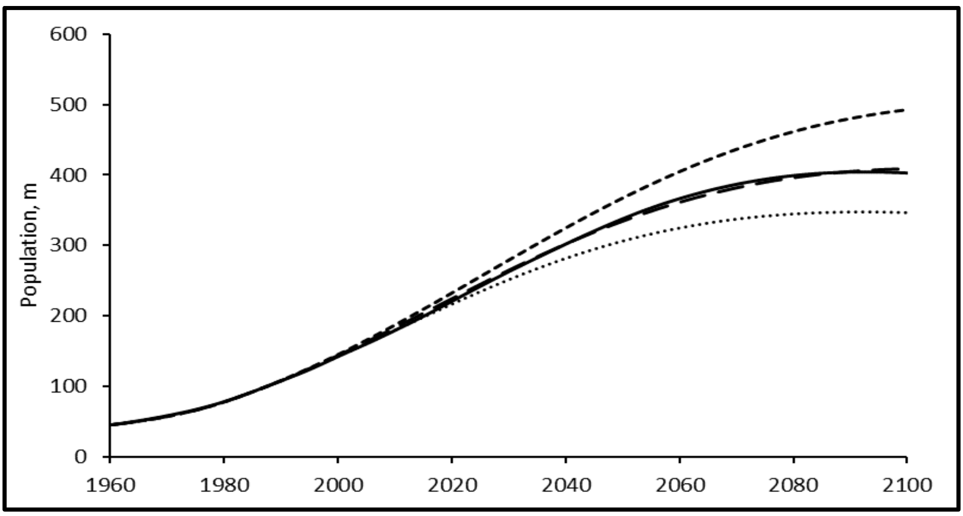


Figure 2. (a). (left) Lorentz curve of cumulative share of income vs cumulative proportion of the population in 2015; (b). (right) Gini coefficient from 1987 to 2015. Based on data in the World Bank Poverty and Equity Database (World Bank, 2020)

Historical and projected population data were obtained from the 2019 World Population Prospects (UN, 2019a). The figures used here are all based on the Median Projection. Figure 3a shows that the population increased dramatically over the historical period, with growth projected to slow and eventually slightly reverse towards the end of the century. The growth rates throughout the period are shown in Figure 3b. The population growth rate rose from 1960 to about 1990, in response to falling death rates, particularly in the early childhood years. Since about 1990, it has fallen in response to declining birth rates. The fall is projected to continue to the end of the century, by which time (according to the median projection) rising deaths in the growing aged population will begin to outnumber births, and the population will start to fall (that is, the growth rate becomes slightly negative).



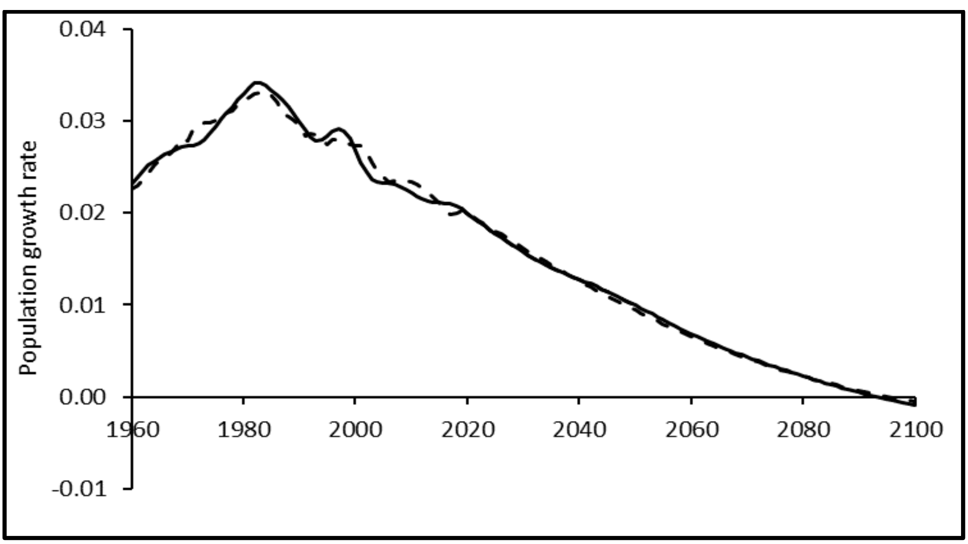
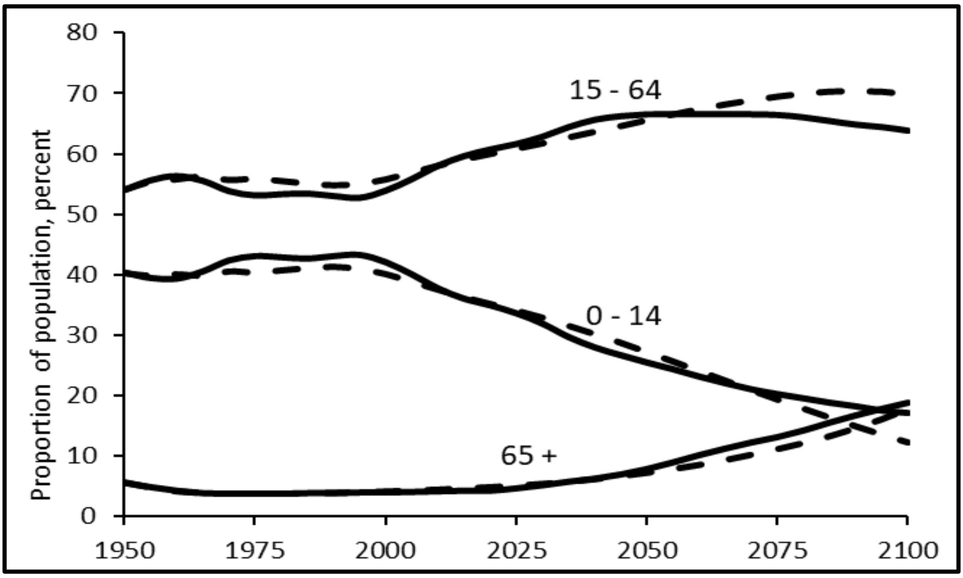


Figure 3. (a) (top) population. The solid line is based on the data from the UN (2019a). The dashed lines are the results of simulations using the simple model (see results section for explanation). (b) (bottom) population annual growth rate. The solid line is based on data from the UN (2019a). The dashed line results from a line fitted to the population annual growth rate as a function of per capita GDP (see section 3.1 for an explanation).

Data for the population in different age groups (available from the World Population Prospects database in five-year increments) were used to construct Figure 4a, which shows the share of the population of young dependents, working age, and aged dependents. Figure 4b shows the dependency ratio.



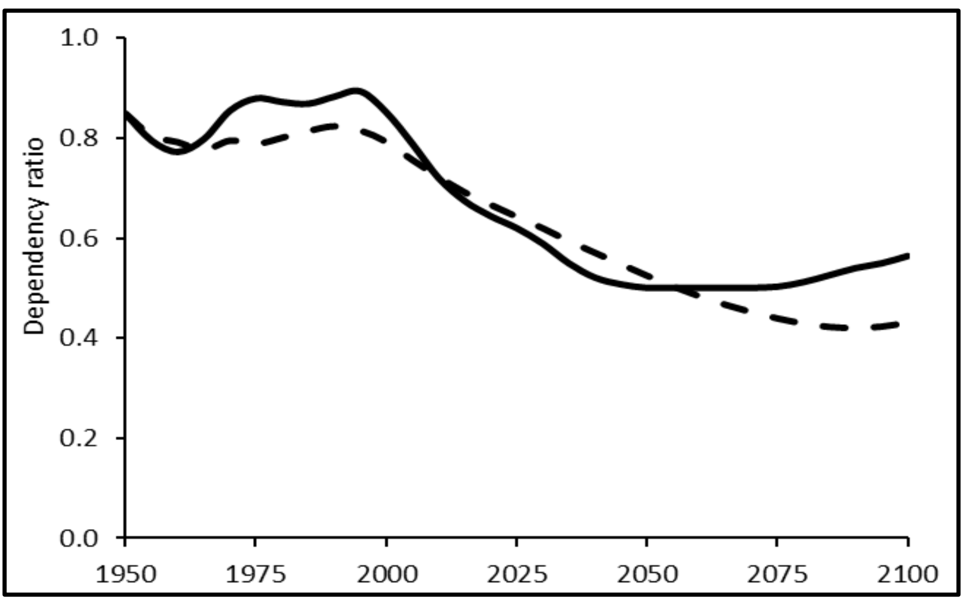


Figure 4. (a) (top) proportion of the population below working age (0–14), at working age (15–64), and older than working age (65+), b. dependency ratio. The solid lines are based on the data from the UN (2019a). The dashed lines are the results of simulations using the demographic dividend model (see section 3.3 for explanation).

3. POPULATION GROWTH—GDP GROWTH MODELS

As noted in the Introduction, the models developed here are based on the unified growth theory models of Galor (2005, 2010, 2011). The models use a dynamic, time-stepping approach, in which the values of the variables (population, GDP, factor growth rates, and other variables) at one-time step are used to calculate the values of the variables in the next time step. Applying the scheme over many time steps simulates the trajectory of population, GDP, and other variables over the long run.

3.1. Simple Model

In the simple model, a Cobb-Douglas production function is assumed in the form:

$$Y = A\,K^{0.5}L^{0.5} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (1)$$

where Y is output (i.e. GDP), A is total factor productivity, K is capital inputs and L is labour inputs. Capital and labour are assumed to have constant returns to scale, indicated by the exponents of the two factors summing to one. Here, I use the production function in an incremental form:

$$\frac{GDP_{t+\Delta t}}{GDP_t} = \frac{A_{t+\Delta t}}{A_t}\,\frac{K^{0.5}_{t+\Delta t}}{K^{0.5}_t}\,\frac{L^{0.5}_{t+\Delta t}}{L^{0.5}_t} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (2)$$

hence

$$GDP_{t+\Delta t} = GDP_t\,A'K'L' \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (3)$$

where the GDP at time $t + \Delta t$ is evaluated from the GDP at time t, and the increment over the time step of the total factor productivity (A'), capital (K'), and labour (L') inputs. In the case of A' and K', these are constants a little greater than 1, representing the annual growth in total factor productivity and capital.

$L' = L^{0.5}_{t+\Delta t}/L^{0.5}_t$ is calculated from the labour input at times t and $t + \Delta t$. The labour inputs are assumed to be in direct proportion to the total population, and hence

$$L' = 1 + PGR_t \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (4)$$

where PGR_t is the annual population growth rate. As shown in Figure 3b, the annual population growth rate varies with time. Here, I assume that the population growth rate results from increasing wealth per capita. That is, with rising per capita wealth, death rates start to fall, perhaps as a result of better nutrition and health care. Then, with further increases in per capita wealth, birth rates fall, perhaps as a result of household preferences as suggested by Galor (2005, 2010, 2011). Figure 5 shows the population growth rate of Figure 3 replotted as a function of wealth per capita (itself calculated from the GDP data of Figure 1 and the population data of Figure 3a).

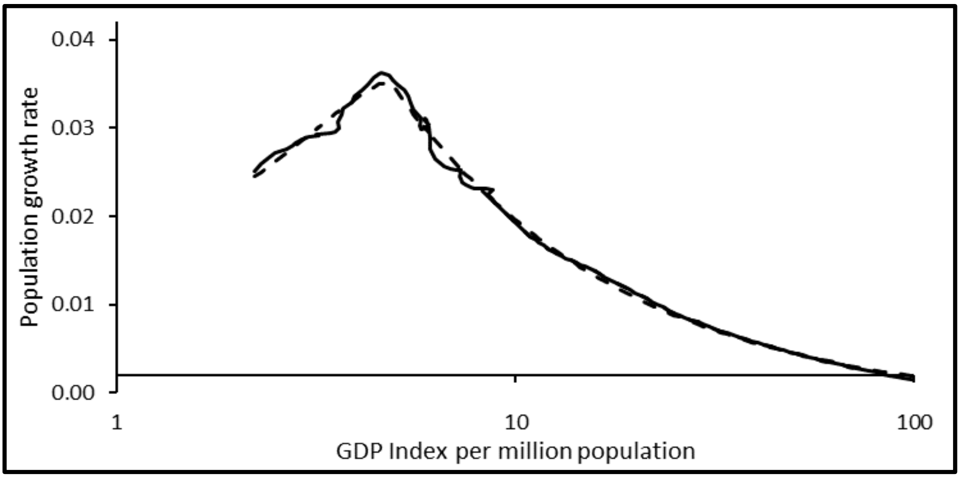


Figure 5. Population growth rate as a function of GDP per capita, where GDP is given as an index (with 1960 = 100) divided by the population in millions. The solid line is based on the GDP data in Figure 1 and the population data in Figure 3a. The dotted line is a fitted function (see text for explanation).

The fitted function shown in Figure 5 is a pair of straight lines in log (PGR') – log (GPD/capita) space (one for the rising part of the curve, the other for the falling part), but with $PGR = (PGR' - PGR_o)$ where PGR_o is a small offset to allow PGR to take on negative values.

Finally, the population, P, at time $t + \Delta t$ is evaluated from the population at time t,

$$P_{t+\Delta t} = P_t\,(1 + PGR_t) \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (5)$$

Note that in this model (and in the two models that follow), GDP and population co-evolve. Causality, in modelling terms, is bi-directional and iterative.

To evaluate the model as specified in Equations (3) and (5), the GDP in 1960 is set at an index value of 100, and the population in 1960 is set at the empirically observed value given by the UN (2019a). The population growth rate uses the fitted curve shown in Figure 5, and I selected the values of A' and K' by trial and error to produce simulated results that are a good match to the historical data and independent projections shown in Figures 1 and 3. Of course, numerical fitting algorithms could be used in place of trial and error.

3.2. Inequality Model

The inequality model is very similar to the simple model, except that the population growth rate as a function of wealth per capita is applied separately to each percentile of the population. At each time in the simulation, the share of wealth in each percentile of the population is calculated from the GDP and the function fitted to the Lorentz curve in Figure 2. This is then divided by the population size in each percentile to give the wealth per capita. A function of the same form as that fitted in Figure 5 (and used in the simple model), but with slightly different coefficients, is then used to calculate the population growth rate separately for each percentile of the population, and the results are summed to give the overall population growth.

The effect of this model is that each percentile of the population moves independently along a population growth curve similar to that in Figure 5. The wealthier groups move early to fewer children and smaller family sizes, and the poorer groups move later.

3.3. Demographic Dividend Model

The final model is based on the modelling of the population dynamics of different age groups. The age groups are those for which the UN (2019a) gives birth rate and death rates; namely, 0-4, 5-9, 10-14, 15-19, 20-49, 50-64, and 65+. The data are given for five-year periods, so in this model, I used a time step of five years, rather than the time step of one year in the simple and inequality models. As a consequence, the input factors A' and K' are raised to the power of 5, so that an annual growth rate of 3 percent becomes a five-year growth rate of about 16 percent, for example.

In each five-year period, the population of an age group is the population in the previous period plus new entrants less exits. Entrants are births for the 0–4 age group, or the whole of the next younger age group less deaths in that group. Exits are deaths in each group. The birth rates as a function of time are transformed into a function of wealth per capita in the same manner as the transformation of the overall population growth rate, as described in section 3.1 above. The result is shown in Figure 6. A two-part function is fitted to the data, with a linear function up to a GDP per capita of about 5.5, and a log-log function for larger values. The best-fit function slightly underestimates the birth rate at high values of GDP per capita.

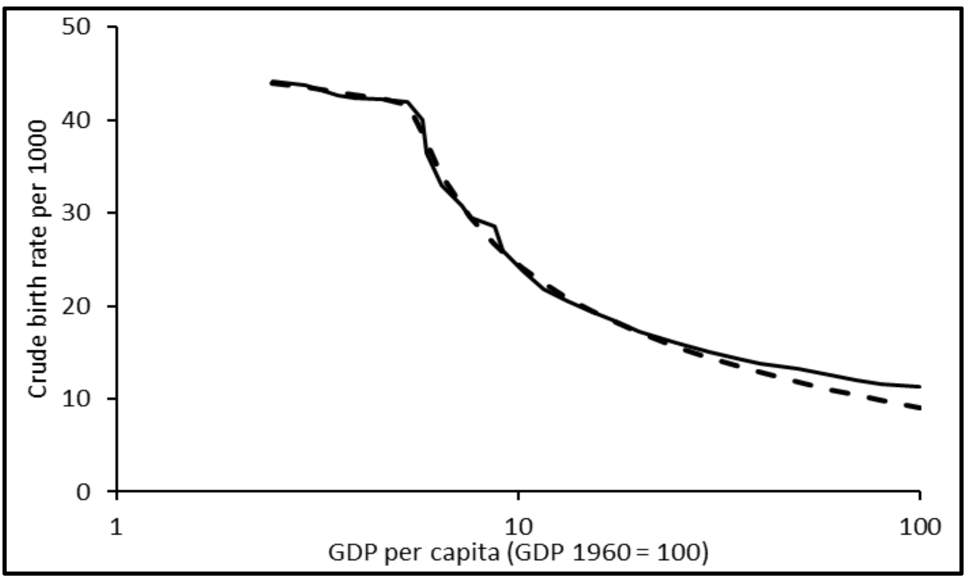


Figure 6. Crude birth rate as a function of GDP per capita, where GDP is given as an index (with 1960 = 100) divided by the population in millions. The solid line is based on the GDP data in Figure 1 and the birth rate data from the UN Population Division (2109). The dotted line is a fitted function (see text for explanation).

The death rates also must be estimated in the model for each population group. In principle, this could be done by fitting functions as with the birth rates. However, this would lead to several functions with a large number of adjustable parameters. For simplicity, I used the deaths per age group, transformed from a time-based to GDP per capita-based form, as a lookup table from which values were interpolated in the model as required.

The above procedures give the population dynamics, including the size of the working-age population and the dependency ratio (which equals the ratio of the 0–14 plus 65+ populations to the 15–64 population). These results were employed in Equation (3) in two ways:

(1) The working age population effect: the labour input term is modified from $\boldsymbol{L' = L^{0.5}_{t+\Delta t}/L^{0.5}_{t}}$ *to* $\boldsymbol{L' = WAP^{0.5}_{t+\Delta t}/WAP^{0.5}_{t}}$*, where WAP is the working-age population.*

(2) The dependency ratio or added productivity effect. With fewer dependents, the working-age population in principle has greater time to deploy on work and greater disposable income to deploy on things such as investments and education. Any of these effects will cause one or more of the total factor productivity (A'), capital (K') and labour (L') inputs to grow at a faster rate than they would in the absence of the demographic dividend. There are a number of ways in which this could be modelled. I chose one expression for illustrative purposes, without implying that it is the correct or only explanation. The capital input term in equation (3) is modified from $\boldsymbol{K' = K^{0.5}_{t+\Delta t}/K^{0.5}_{t}}$ to $\boldsymbol{K' = K^{0.5/DR\wedge 0.04}_{t+\Delta t}/K^{0.5/DR\wedge 0.04}_{t}}$, where *DR* is the dependency ratio. Since the dependency ratio is less than 1, this expression has the effect of slightly increasing the rate of growth resulting from capital inputs. Furthermore, K' is no longer a constant but evolves with the changing dependency ratio.

3.4. Scenarios

The three models above were each deployed to simulate illustrative scenarios, three each in the case of the simple and inequality models, and two for the demographic dividend model. The scenarios were all assessed to 2100; for some long-term planning, such as for water resources, a period of this order is useful

(1) Simple Model: (i) a simulation that results in a good fit to the observed and projected GDP and population growth from 1960 to 2100; (ii) a simulation in which the population grows slightly faster during the second, slowing growth phase, than is given by the function fitted in Figure 5; and (iii) a simulation in which the population grows slightly slower during the second, slowing growth phase than is given by the function fitted in Figure 5. The faster and slower population growth was achieved by changing the slope of the falling part of the log (*PGR'*) – log (GPD/capita) curve to be 11 percent less and 11 percent more than in the good-fit simulation.

(2) Inequality Model: (i) a simulation that results in a good fit to the observed and projected GDP and population growth from 1960 to 2100; (ii) a simulation in

which the wealth per capita is distributed more evenly with the passing of time than is given by the Lorentz curve in Figure 2a; and (iii) a simulation in which the wealth per capita is distributed less evenly with the passing of time than is given by the Lorentz curve in Figure 2a. The second and third scenarios start with the distribution given in Figure 2a, which has a Gini coefficient of 0.31, and evolve linearly with time to new distributions. In the second scenario, the Gini coefficient finishes in 2100 at 0.18, and in the third, it finishes at 0.45.

(3) Demographic dividend model: (i) a simulation that results in a good fit to the observed and projected GDP and population growth from 1960 to 2100, using only the working age population effect in 3.3; and (ii) a simulation in which GDP and population grow according to both the population effect and the dependency ratio effect in 3.3.

4. RESULTS

4.1. Simple Model Simulations

The results of a simple model simulation that is a good fit to the observed historical and projected future GDP and population is shown in Figure 1 (GDP) and Figure 3a (population). The fits overall are very good. However, the observed GDP grows faster than is simulated from 1960 to about 1990, and then slower from about 1990 to 2020. I shall return to this point in the discussion section below but, briefly, it is likely to be a result of factors other than the growth in capital and labour, and the assumed constant total factor productivity growth. The growth of GDP, population, and GDP per capita from 1960 to 2100 is shown in Table 1. Overall, the GDP is simulated to have increased 470 times from 1960 to 2100, with the population increasing 9.1 times and GDP per capita increasing 51.7 times.

Table 1

GDP, Population and GDP per Capita in 1960 (all Models) and Simulated in 2100

Model / Simulation / Year	GDP (1960 = 100)	Population (m)	GDP / Capita
All Models, Values in 1960	100	45.0	2.22
Simple Model, Values in 2100			
Good Fit	46,979	409	114.9
Faster Population Growth	51,565	493	104.7
Slower Population Growth	43,262	347	124.8
Inequality Model, Values in 2100			
Good Fit (Gini 2100 = 0.31)	46,988	409	114.9
More Equal (Gini 2100 = 0.18)	43,947	358	122.8
Less Equal (Gini 2100 = 0.45)	50,195	467	107.5
Demographic Dividend Model, Values in 2100			
No Dependency Ratio Effect	50,961	409	126.2
Dependency Ratio Effect	53,240	409	134.8

The two scenario simulations with faster and slower population growth result in the populations shown in Figure 3a. The population in 2100 of 347 m (slower population growth scenario) or 493 million (higher population growth scenario) are well within the upper and lower 80 percent probabilistic bounds (of 282 and 571 m) of UN (2019a) using the probabilistic method (UN, 2019b). The faster population growth scenario results in a higher GDP through the labour input effect, but in a GDP per capita about 9 percent lower than in the good-fit case. The slower population growth scenario results in lower GDP, but GDP per capita is about 8.7 percent higher than in the good-fit case.

4.2. Inequality Model Simulations

The results of the inequality model simulation that is a good fit to the observed historical and projected future GDP and population are barely distinguishable from the simple model good-fit case (Table 1). The simulation with greater equality leads to a lower population because the population growth rate falls more rapidly with greater equality, as the poorest people gain wealth more rapidly in this scenario. This in turn leads to a smaller GDP, but a larger GDP per capita (Table 1), similar to the slower population growth scenario of the simple model. Conversely, the simulation with less equality leads to a higher population, and higher GDP but lower GDP per capita (Table 1). Again, this is similar to the simple model with faster population growth. However, the degree of change to inequality required to achieve this similarity, as indicated by the Gini coefficients of 0.18 (low population case with much-reduced inequality) and 0.45 (high population case with much increased inequality) are outside all historical experience (cf Figure 2b) and should be considered unlikely to obtain in practice.

4.3. Demographic Dividend Model Simulations

The demographic dividend model simulation with no dependency ratio effect results in a population in 2100 similar to that of the good-fit simple model, but with GDP and GDP per capita similar to the faster population growth simple model scenario. This is because the working-age population grows faster than the population overall. The addition of an additional effect from savings as an investment in capital leads to even faster GDP and GDP per capita growth, without impact on population growth. The population dynamics simulated for the young, working-age, and aged population age groups are shown in Figure 4a, and the resulting simulated dependency ratio is shown in Figure 4b. The model simulates the observed and projected population dynamics reasonably well.

5. DISCUSSION

The results outlined above show that the models can simulate the co-evolution of population and GDP with varying population growth, varying inequality, and varying population age-group dynamics producing a demographic dividend effect. While the scenario simulations do not include cases where the total factor productivity and/or capital input factors are varied, incorporating such variation would be straightforward.

The models simulate the observed and projected population and GDP growth reasonably well. However, they do not perfectly match the observed historical GDP. As shown in Figure 1, the models simulate faster GDP growth from 1960 to about 1990 and slower growth from about 1990 to 2020, resulting from the increasing rate of population growth in the first period and the decreasing population growth thereafter. Nevertheless, the observed GDP growth in the first period is faster than that simulated, and it is slower than simulated in the second period. GDP growth in Pakistan is influenced by many other factors, including a slowing of total factor productivity (López-Cálix, et al. 2012; Amjad & Awais, 2016; Saleem, et al. 2019), a decline in capital accumulation (López-Cálix, et al. 2012), constraints on growth due to periods of poor foreign exchange reserves (Amjad, 2014), increased terrorism (Sami & Khattak, 2017), and foreign direct investment and remittances (Ullah, et al. 2014; Tahir, et al. 2015).

Incorporating some or all of the other factors into the production function model for GDP growth would not be difficult. However, it is not the purpose of this article to test explanations of GDP growth in Pakistan. Rather, the purpose here is to develop models for the effect of population growth, poverty, and population dynamics. A model that incorporates the full range of factors would obscure the population effects.

The relationship between population growth and economic growth is controversial, with some authors arguing that population growth enhances economic growth, while others conclude the opposite (as reviewed by Peterson, 2017). Peterson (2017) notes that a high economic growth rate may be associated with a high population growth rate, but this will not necessarily produce growth in per capita GDP. The simple model scenarios resulted in lower population growth associated with higher growth in per capita GDP, and vice-versa. The scenarios assumed independence of per capita economic growth and population growth; more precisely, the total factor productivity and capital input growth effects in Equation (3) were assumed not to be affected by population growth. Peterson (2017) notes that this is not necessarily the case. As noted above, the total factor productivity has slowed in Pakistan (López-Cálix, et al. 2012; Amjad & Awais, 2016; Saleem, et al. 2019), and the population growth rate has slowed over the same period (as shown in Figure 3b). Testing alternative assumptions about total factor productivity and capital input growth effects would be straightforward with the simple model (and with the two other models).

The inequality model incorporates the overall population growth effect of the simple model, and for a base case in which inequality is set at the historically observed level, it produces results very similar to those of the simple model. As noted above, however, the similarity is achieved with assumptions about the degree of change in inequality that are outside historical experience and are unlikely in practice. The inequality model then adds the association between income level and family size (noted in Pakistan by Qureshi & Arif, 2001; ADB, 2002; Hyder, et al. 2010; Arif, 2013; Majeed & Malik, 2015; Ibrahim, et al. 2019; Baulch & McCulloch, 1998) postulated as an association between population growth rate and income. This model can be used to examine the implications of inequality on population and GDP growth (cf. Shahbaz, et al. 2013; Soharwardi, et al. 2018).

The third model introduces the impact of the changing age structure of the population in Pakistan, and hence the prospect of a demographic dividend. The results of this model are consistent with the observations of Amjad (2013) that the onset of a demographic transition has as yet limited impact on economic growth. However, consistent with Choudry & Elhorst (2010) and Iqbal (2015), the impact may be greater in the future.

While it is not the purpose of this paper to assess or explain future projections of population and GDP, a few remarks may nevertheless be made. As noted in the Introduction, In the decade to 2020, Pakistan's population grew at a little more than 2 percent per year, and its gross domestic product (GDP) grew at about 4 percent per year. In terms of Equation (3), the contribution to GDP growth of the labour input term (L') for the simple and inequality models is about $1.02^{0.5}$, or about 1 percent per year. The other approximately 3 percent per year results from the combined total factor productivity (A') and capital (K') input terms. By 2100, the population according to UN (2019a) will have just started to shrink, so under the assumption that A' and K' remain constant, the GDP growth by 2100 for the simple and inequality models will be around 3 percent per year. The demographic dividend model based on the working age population effect alone produces a similar result for GDP growth at 2100. However, if there is also a dividend resulting from the lesser dependency ratio boosting GDP growth (perhaps through savings as investment in capital, used here as an illustrative example), the GDP growth will be greater (under the assumption that A' and K' remain constant). Given these competing influences, the projection of 4.1 percent annual growth in GDP assumed here (and broadly consistent with PWC, 2017) is not unreasonable. However, as noted by Bloom, et al. (2011) and Bongaarts, et al. (2013), capturing the benefits of the demographic dividend is not automatic but requires the right policies. The UN population projections, while they recognise that socio-economic factors influence population growth, do not explicitly use them in the calculations (UN, 2019b).

Since all three models have produced broadly similar results, they are not suited to discriminating amongst explanations of the links between population growth and GDP growth. Their purpose, as noted, is to test assumptions and influences on broader issues of long-term planning, such as food and water security.

The unified growth theory of Galor (2005, 2010) seeks to explain human development in the very long run. Galor (2010) discussed the recent demographic transition as resulting in part from the evolution of preference for having fewer offspring, but devoting more resources to the education of those offspring (Galor terms this quality or human capital). The evidence and the models presented here are consistent with this aspect of the unified growth theory. Figure 6 shows a slightly declining birth rate up to a GDP per capita of about 5 (with GDP given as an index with 1960 = 100, divided by the population in millions), after which the birth rate declines steeply. The onset of the steeper decline occurred around 1980–85. Figure 5 shows that in the early period when the birth rate was only slightly declining, the population growth rate was increasing. This is because the death rate was falling rapidly during this period (UN, 2019a). From about 1980–85, the decline in the death rate slowed (because it began to reach lower levels and so had less far remaining to fall), and the birth rate began to fall rapidly, and so Figure 5 shows the reversal of the population growth rate at the same GDP per capita (and hence the same time) as the decline of the birth rate in Figure 6. Evidently, parents started to have fewer offspring around 1980–85, somewhat after death rates had begun to fall. The case of Pakistan and the models derived here are consistent with the unified growth theory idea that with increasing wealth, parents have fewer offspring and invest more in human capital.

Finally, the models presented here shed light on what might be achieved by way of population growth and GDP growth, but they do not tell us how to achieve particular outcomes. The scenarios indicate that lower population growth appears achievable in the sense that the simulated lower population is well within the 80 percent probabilistic bounds of the UN (2019a). If achieved, this would render planning for future resource demands (such as water and food supply) easier, and also be associated with higher per capita GDP. Such a future is consistent with the vision propounded in Vision 2025 (Planning Commission, 2014) and by the World Bank (2019), which suggests that reducing population growth in Pakistan is necessary for economic growth. The scenario analyses suggest that, whatever the merits of reducing inequality in Pakistan, it is alone insufficient to result in substantial reductions in population; as noted above, the degree of change in inequality required is quite unlikely to be realised in practice. The scenarios assessed with the demographic dividend model suggest that there is a substantial opportunity to capture the economic dividend in the coming decades, consistent with the suggestion of Choudry and Elhorst (2010). If this is achieved, there is the prospect of substantial gains in GDP and GDP per capita.

6. CONCLUSIONS

We conclude that the three population—GDP growth models presented here are well suited for their intended purpose, which is to help examine trends in population growth and economic growth. The models simulate the co-evolution of population and GDP as affected by GDP growth input factors, population growth rates, wealth inequality, and population age dynamics leading to a demographic dividend. They are consistent with theories of population and economic development.

Pakistan has the opportunity to change its population growth and GDP growth trajectory. Policies should aim to slow population growth, which in turn will ease future challenges such as planning for water and food security and could also lead to a reduction in poverty through greater GDP per capita. In the coming decades, there will be a higher ratio of workers to dependents. Policies should aim to capture this demographic dividend, by ensuring that the dividend is directed to purposes such as infrastructure investment rather than consumption; if successful, there will be substantial gains in GDP and GDP per capita, again potentially leading to a reduction in poverty. On the other hand, reducing inequality, while undoubtedly desirable on other grounds, appears to be marginal in terms of lowering population growth or raising GDP.

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