A Complete Set of Consumption Coefficients for West Pakistan

by

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INTRODUCTION

I.1

In this paper, a set of coefficients is derived which can be used to project West Pakistan's private consumption in the medium and longer term.

For ten urban and rural income groups separately, income, price and cross elasticities of demand have been calculated. Thus, the methodology permits to trace the influence of changes in incomes, income distribution and prices on consumption.

I.2

Most development models concern themselves with aggregate income effects only, and assume constant income distribution and prices. This may be warranted as long as no structural shifts occur. Insofar as development implies structural shifts, there may be an inconsistency.

Apart from this, the attainment of a more equitable income distribution is more and more being emphasized as an independent goal of socio-economic planning. Changes in distribution are more often than not at least partly the result of changes in relative prices. Obviously, then, both incomes and prices must be included in the planning framework if anything is to be said about the effect of planned development on equity.

Finally, even leaving the equity aspect aside, economic growth is influenced by the income and price structure. So, *e.g.*, the structure of incentives influences production, and the income distribution affects savings. In this context, it is often argued that a skewed income distribution, while undesirable in itself, is necessary to generate the savings required for economic develop-

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ment. However, if consumption by the affluent requires more capital, or import-intensive production, the net effect might not be conducive to growth at all.

I.3

The present paper is a partial result of an effort to include these aspects into a policy model for West Pakistan [6]. In the case of West Pakistan, such an effort seems to be especially called for. In the first place, the so-called "green revolution" opens the possibility of an adjustment of the prices of basic food-grains, and the policy alternatives in this area need to be carefully analyzed with respect to their consequences for equity and growth. In the second place, a number of studies [10; 11; 16; 21] indicate that there presently exist a number of distortions in the Pakistan economy. Given the present paucity of investment resources for West Pakistan, it appears to be eminently worthwhile to try to estimate what influence the gradual elimination of these distortions might have on West Pakistan's growth potential.

I.4

If the structure of consumption is to be analyzed in detail, the number of coefficients to be estimated is substantial. This requires the use of a wholesale method of computation, since otherwise the research time involved would be prohibitive[1]. The starting point was the method developed by Frisch [9]. Given Engel functions and an assumption on the income elasticity of the marginal utility of money, Frisch derives price and cross elasticities under the assumption of want-independence.

This method was extended in two ways:

a) it was applied to different income groups. This was possible only by making assumptions about the relations between income level and its marginal utility. Some suggestions on this relation have been made by Frisch.

b) assumptions about the degree of substitutability replaced those of want-independence in the food group. To assume different food items to be want-independent seems to stretch reality too far. The substitutability assumptions seem to be of more general validity than Barten's [3] but they were not empirically estimated.

The methodology developed in this paper depends on a number of as yet untested quantitative assumptions. Still, it yields quantitative estimates which can be compared with the actual performance of the economy. In the meantime, they at least can give an idea of the orders of magnitude of the coefficients involved.

[1]It took the author more than a month to come up with a reasonable estimate of one price elasticity (*see* [7]).

Because of the methodological aspects and also because the calculation by income group leads to overall results different from those of Frisch (*see* subsection II.2 below), the present study is published separately.

I.5

The further set-up of this paper is as follows. Section II discusses the methodology in more detail. Section III describes the survey data used and the methods followed to obtain Engel curves. Section IV discusses the major additional assumptions and the quantitative estimates involved. Section V gives the main results, and discusses them in some detail. Section VI contains some concluding remarks. Finally, appendices give the relations used to derive the elasticities, and more detailed results.

II. BASIC METHODOLOGY

This section discusses the methodology in general terms. Specific steps in computation are described in subsequent sections. The relationships used in the calculations are derived in Appendix A[2].

II.1 Income-Expenditure Relations

Cross-section data were used for the estimation of income elasticities. This appeared to be warranted, since the object was to estimate longer-term elasticities.

Income and expenditure data per income group are available only per household. Between and also within income groups, households are of varying size. Obviously, this implies variation in real income. Since income-expenditure relations are traditionally mainly based on real income differentials, household data had to be brought on a per capita basis. Given the lack of information, this had to be done in a very crude way. It is hoped that in future a cross-classification of expenditures by income group and household size will become available. Then, the effects of income and household size on consumption could be disentangled in a much more satisfactory manner.

In estimating the relationship between per capita expenditure on specific items and total per capita expenditure, a number of different specifications were tried. Some of these included average household size for the income group in order to catch economies of scale with varying household size.

Through an adjustment of coefficients, household-size effects were made to cancel each other out such that a variation in household size would imply a

[2]Appendix A develops the minimum of relations in the simplest possible manner. For a more complete and rigorous treatment of most aspects, *see* Frisch [9].

different, but still consistent, breakdown of total expenditure over its components.

The influence of household size thus having been eliminated, the relations between specific and total expenditures were studied in detail.

Such a detailed study was necessary, since the object of the exercise was to obtain coefficients by income group. If one is estimating a composite coefficient for the whole income range, the fact that the relation chosen does not fit well for a part of that range does not need to disturb one very much. In this case, however, it was important to obtain as good a fit as possible over the whole income range. As was to be expected, it was found that simple linear, semi-logarithmic or double-logarithmic specifications were not able to catch all the information contained in the data. On the other hand, quadratic or cubic relations often implied unrealistic extrapolations on one or both sides of the income scale. Finally, statistically acceptable but different specifications led to sometimes quite different results[3], and the best relations for the different items of expenditure did not add up to the total.

The detailed study of the relations led to the adoption of smooth hand-drawn curves which combine the best statistical results, and even try to improve on them where possible. Such a procedure seems to be decidedly superior to one in which regression results are accepted uncritically, and corrected in some mechanical way to force them into a consistent pattern.

The Engel curves thus derived were combined with savings functions and income distribution data to yield income elasticities per income group, and weighted elasticities for main categories of incomes.

II.2 Price Elasticities

Price and cross elasticities have been estimated by making certain assumptions about the utility function of the "average consumer". The alternative approach would be to estimate price elasticities from time series. However, this road seems inaccessible because consumption data are generally unavailable. Data on apparent availability may be substituted, but changes in stocks may then easily disturb the picture (example is given in [7]). If one wants to correct for this, a careful analysis is necessary in every case, and a large investment in research will have to be made. Eventually, some of this effort will be necessary in any case to test the results obtained in this paper.

[3]Thus, at an income level of 30 rupees per month in the rural sector, the marginal expenditure on food varied between 40 and 49 per cent and at 90 rupees per month in the urban sector, the range of marginal expenditure for services was from 36 to 67 per cent.

II.2.1 The case of want-independence: If one assumes the individual commodities to be want-independent of each other, the derivation of price and cross elasticities is fairly straightforward, given certain assumptions about the income elasticity of the marginal utility of money (income). The methodology has been developed by Frisch [9], and its main outlines have been sketched in Appendix A to this paper[4].

Except for the income-expenditure relations already derived, the method requires a quantitative estimate of the elasticity of the marginal utility of money (Z). Empirical estimates of this variable have been made for a number of countries, but always for the "average consumer". If, in this paper, elasticities are to be estimated *by income group*, the *form* of the Z-function also has to be specified. This problem is discussed in subsection IV.1 below. Two alternative specifications of the Z-function have been utilized in this paper. They both assume a high elasticity[5] at low incomes, declining as incomes rise.

After price elasticity coefficients per income group are derived, they can be weighted over income groups to arrive at overall coefficients. This approach leads to results which are different from those that would have been obtained if the calculations had been based on average throughout[6]. Substitution effects come out lower for goods with a low income elasticity (and vice versa). The reason is that a larger proportion of those goods is consumed by the lower income classes which, given their higher Z, have a lower propensity to substitute. Thus, for wheat, with a low income elasticity, the substitution part of the price elasticity comes out at —.05 instead of —.10. For domestic help which is highly elastic, the results are —1.48 and —1.15, respectively.

Although, given the uncertainties throughout, the magnitude of these differences should perhaps not be overemphasized, they imply that it is methodologically unsound to derive price and cross elasticities under want-independence from income elasticities and one (independently estimated) price elasticity. In principle, the price and income elasticities for one good will yield a weighted Z for that good. However, if Z is different between income groups, the Z-function is not fully determined by one observation. Given stochastic disturbances, a number of observations (covering goods with sufficient variation in

[4]From a methodological viewpoint, the method is open to criticism, since it implies the assumption of cardinal utility. However, to quote Frisch : "From assumptions as to want-independence there follow very definite conclusions about certain *observable* demand phenomena. Assumptions of this sort can, therefore, be classified as refutable hypothesis. To proceed from assumptions about an abstract theoretical set-up and from them to draw conclusions about the observable world and to test... whether the conformity is "good" enough, is indeed the time honoured procedure that all empirical sciences... have used. I shall, therefore, not plead guilty of heresy.."[9 p. 178].

[5]Throughout this paper, an elasticity is termed "higher", the more it differs from 0 (whether in positive or negative direction).

[6]This point appears to have been overlooked by Frisch and by others who have used his method.

income elasticity) will be necessary to estimate both level and form of the Z-function.

II.2.2 The case of want-dependence: The assumption of want-independence is very convenient and will often be in reasonable conformity with reality. So, between broad groups of expenditure for, *e.g.*, food, clothing, housing and services, the assumption appears to be justified. Even within those groups, some goods satisfy rather separate wants like medical care, domestic help and education within the services group. For the present study, the five main expenditure groups (*see* Appendix C) and the subcategories within four of these five are assumed to be want-independent. However, this assumption would go too far within the food group, which accounts for about 55 per cent of all expenditure.

The method that was developed to handle want-dependence is explained in subsection A.2 of Appendix A. In it, the extent to which the marginal utility of money spent on good k is affected by additional expenditure on good i, is linked to the effect that additional expenditure on either good has on its own marginal utility. The link is made through a substitutability factor x_{ik} which can vary from +1 (complete substitutability, goods are identical to the consumer) through 0 (goods are completely want-independent) to —1(goods are completely complementary). In practice, no complementarity has been assumed, and the maximum substitutability allowed has been .4 (*see* subsction IV.2 below).

III. MAIN DATA INPUTS AND DERIVATION OF ENGEL CURVES

III.1 Data Inputs

The main data source was the *Quarterly Survey of Current Economic Conditions in Pakistan, 1963/64* [18] (QSCEC). Although these data are not very recent, they offer the great advantage that work on income distribution and savings has been based on them. Since expenditure patterns do not change rapidly, a detailed analysis of this survey still seemed worthwhile.

The sample consisted of about 2100 rural and 1700 urban households. They were divided over 11 income groups. Roughly 70 per cent of all households belonged to the second to fifth income groups. The upper groups, especially in the rural sector, contained relatively few observations.

Total expenditures were divided over 31 items, of which three small ones were grouped together for this study.

A complete commodity code is given in Appendix C.

As pointed out earlier, an important methodological drawback is that the grouping criterion has been the household income and not per capita income. Given a substantial variance in household size within each income group, this

implies differences in economic welfare within each group with probable consequences for its average expenditure pattern.

Another less desirable feature of the data is the great importance of the "All other miscellaneous" expenditure category, especially in the rural sector. Although small at low income, its importance increases rapidly. In other words, in marginal rural expenditure, the unspecified part is large (25-30 per cent, compare Table B-6) and the data are, therefore, incomplete. One suspects that the unspecified part is largely spent on services and social obligations.

Apart from these points, the usual reservations on such survey data are in order. Still, the analysis shows that, as a whole, the data on the expenditure side are reasonably consistent. There is, therefore, no reason to doubt their basic validity.

On the income and savings side, Bergan [5] has analyzed the QSCEC data. He has made a number of corrections on incomes, eliminating certain items that should not be classified as income, and adding income at the upper end of the scale on the basis of income-tax data. After that, he has estimated the income distribution and has made an overall check on savings. This work has been of great use for the calculations in this paper.

III. 2 Income Distribution and Savings

Since Bergan's income-distribution data were based on households, a transformation was necessary to convert them in per capita terms. The QSCEC publication indeed gives the household size distribution per household income group.

However, it is impossible to regroup the expenditure data in a similar fashion. Conversion of incomes and expenditures in a dissimilar manner would have led to inconsistencies. Therefore, the close empirical relationship between average income and household size (per income group) was used to estimate an average household size for each level of household income. Division yielded a detailed per capita income distribution.

This distribution was consolidated into ten income groups with averages 20 per cent apart (the uppermost income group twice this distance). The results are given in Table I. For the presentation of the income elasticities, further consolidation into L(ower), M(iddle) and H(igher) income classes has taken place. These classes include income groups 1-3, 4-6 and 7-10, respectively.

On savings, Bergan has presented no data per income group. Overall, his calculations yielded a gross domestic savings rate which was about 2 percentage points higher than the Planning Commission's. Assuming that the truth lies in the middle and applying the correction equally to West Pakistan rural

TABLE I

PER CAPITA INCOME DISTRIBUTION AND SAVINGS

Income group No.	Average income	Income distribution			Savings	
		Rural	Urban	Total	Rural	Urban
	(*Rs. per month*)	(.....*per cent*.....)			(*Rs. per month*)	
1	20.30	11.0	.9	8.2	—.12	—.46
2	24.40	15.3	13.5	14.8	.70	—.13
3	29.30	16.5	15.6	16.3	1.49	.27
4	35.20	18.6	12.5	16.9	2.81	.81
5	42.20	14.5	9.3	13.0	4.54	1.50
6	50.60	9.4	8.7	9.2	6.93	2.46
7	60.80	6.1	7.3	6.4	10.07	4.18
8	72.90	4.4	5.5	4.7	14.11	6.32
9	87.50	1.6	5.2	2.6	19.29	9.77
10	126.00	2.6	21.5	7.9	32.74	20.10

and urban personal savings, as calculated by Bergan, the average rural and urban savings rates come out at 8.4 and 6.0 per cent, respectively.

The savings rates implied by the QSCEC data are much higher. Bergan has arrived at his rates by deducting certain items which should not have been included in income. The distribution of these deductions over income groups is, however, not known.

For the present study, savings functions have been estimated by accepting the average savings rates quoted above, assuming slightly negative savings rates by the lowest income groups, and applying upper bounds for marginal savings (30 per cent for urban, 35 per cent for rural). It is clear that the empirical basis of these savings relations is limited. Still, they appear to be "reasonable". They are presented in Table I.

The substantial difference between rural and urban savings rates may reflect differences in attitude. It is also possible that urban costs of living are higher. So, *e.g.*, expenditures for rent are considerably higher in urban than in rural areas.

III.3 The Expenditure Relations

III.3.1 The influence of household size : The basic data from the QSCEC, 1963/64, on expenditure per category are again in terms of households. Given the close positive correlation between household size and income, straight correlation would have severely biased the results (bringing all elasticities closer to unity[7]). All expenditures were, therefore, brought on a per capita basis, per income group.

Clearly, this may not wholly solve the problem, since the relative age and sex distribution of the households in the different income groups may not be the same. The QSCEC did not include data on household composition. The earlier National Sample Surveys of 1960 and 1961, however, did and these data indicate only small differences in age composition over income groups. On this basis, the assumption has been made that the relative household composition in the QSCEC data was uniform[8].

Although an analysis in terms of per capita rather than per household terms needs to be made if the income coefficients are to be measured accurately, this does not imply that household size can be disregarded in explaining the consumption behaviour. Obviously, the size of the household may influence the consumption pattern through economies and diseconomies of scale. Therefore, household size was introduced as a separate explanatory variable in some of the expenditure relations that were tested. Short of a direct analysis of the relation between household size and expenditure within each income group (for which the data are lacking) this seems to be the best procedure to disentangle income- and household-size effects.

The specifications that were tested by regression analysis[9] were linear, square, cubic, semi-logarithmic and double-logarithmic relations between expenditure on the particular item and total expenditures. All relations were tried with and without a household-size term[10].

The results for the urban sector are given in Appendix Table B-1. For 13 commodities, statistically acceptable relations which included a household-size influence were found. However, for some of these commodities, other relations performed even better, or the household-size influence seemed to have

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[7]An analysis of the expenditure pattern in terms of households would be relavant only if it could be assumed that the relation between household income and size *over time* would be the same as presently *between households*. This seems to be an unwarranted assumption.

[8]This may not be quite realistic for the smallest households, which presumably have a relatively higher number of adults.

[9]The observations were weighted by the square root of the number of households sampled in the income group.

[10]Attempts to allow for a changing household-size elasticity by introducing a log term, and by estimating coefficients for small and large households separately, were not successful.

the wrong sign. Eventually, six items remained. For those items, the coefficients were slightly adjusted to obtain a consistent estimate. The results for three representative urban income groups are given in Appendix Table B-2.

In a money sense, the most important economy of scale exists in rents, per capita expenditure on house rent for small households is about 20 per cent above "normal" and for large households 20 per cent below. A similar but smaller effect can be observed in the fuel and light category. Also, larger families seem to spend less on tobacco.

As against this, per capita expenditures on transport go up with increasing household size presumably because then not everybody can live near his work. The other offsetting elements are milk products, meat and fish. These items are probably truly "offsetting" in that they reflect the somewhat greater affluence of the larger households obtained through the economies of scale that have been noted.

For the rural sector, similar calculations were performed. Fewer significant household-size effects were found only in rents and transport. Quantitatively, they were also somewhat smaller than in the urban sector.

III.3.2 The Engel curves: After the elimination of the household-size influence, the expenditure relations given in Appendix Table B-1 were further explored. The methodology has been described in subsection II.1 above. The technique was to allow only gradual changes in marginal expenditure except in a few cases where heterogeneity of the expenditure category was evident.

The average and marginal coefficients which resulted have been given in Appendix Tables B-3 to B-6.

IV. QUANTIFICATION OF OTHER VARIABLES

IV·1 The Income Elasticity of the Marginal Utility of Money

An important input into the system of equations is the elasticity of the marginal utility of money Z. Both the *level* and the *form* of the Z-function need to be known.

As to the level, no Pakistani data are available and no empirical estimate will be attempted here. However since Frisch's article, an average Z for the whole economy has been estimated for a number of countries [1; 2; 4; 8; 14; 17; 20]. All estimates for developed countries lie in the range between -1.5 and -3. The smallest ones are for the United States and Canada; for European countries, like Norway and the Netherlands and for Australia, values from -2 to -3 were found. Between countries, Z may, thus, be negatively related to income, but the evidence for this seems to be inconclusive. The fact that within a country

one will expect such a relation (*see* below) also does not necessarily imply that the same should be true between countries. Finally, for some developing countries in Latin America, low values of Z were found [8 ; 17].

In this paper, two alternative Z-functions have been used. The first has a weighted average of −2.25 and, thus, lies in the middle of the range cited above. The second indeed gives some credence to the argument that poorer countries, like poorer individuals, might have a higher Z. Its mean value is −3.35.

As regards the form of the Z-function, no research results, even for other countries, can be cited. Frisch [9] has offered a suggestion which is, however, not the result of an empirical investigation. He suggests values of −10 for "an extremely poor and apathetic part of the population" through −2 for "the middle income bracket" to −.1 for "the rich part of the population with ambitions towards conspicuous consumption".

It indeed appears reasonable to assume that at the bottom end of the income scale, Z will be relatively high: if one is near starvation, the first addition to income will be much more valuable than the next, *etc.* On the other hand, the suggestion made for the rich seems rather questionable. For a want-independent good with an income elasticity of unity, a Z of −.1 would imply the very high price elasticity of −10 (Appendix A, relation (18)). It seems to be in conflict with everyday observable reality to assume that the very rich react violently to small price changes. Therefore, it is also unrealistic to assume that Z falls to such a very low level within the range of incomes that can be observed in practice.

If one accepts the above constraints for both ends of the curve, if the function is assumed to be monotonic, and if its average is given, then its form is determined within fairly narrow limits. The form chosen was a hyperbolic one, since this indeed has the desired characteristics of a rapid decrease at low incomes, and of a flattening out thereafter. For Z_1, the horizontal asymptote is 0, but for few individuals in Pakistan the Z would fall below −.5. For Z_2, the horizontal asymptote has been put at −.5; thus, Z will never fall below this level. The specific functions and their values at the different income levels are given in Table II.

IV.2 The Matrix of Substitutability Coefficients

The substitutability coefficients, as defined in paragraph II.2.2 and Appendix subsection A.4, are another important input into the system.

Of the 16 items in the food group, 3 are considered to be want-independent: condiments and spices, tea and tobacco. For the remaining 13, a matrix of substitutability coefficients has to be estimated. The problem has two aspects, the structure and the level of substitutability.

TABLE II

ASSUMED VALUES FOR THE INCOME ELASTICITY OF THE MARGINAL UTILITY OF MONEY Z

Income group	Monthly income	Value of $Z_1 = \frac{-100}{Y}$	Value of $Z_2 = \frac{-120}{Y} - .5$
	(*Rs.*)		
1	20.30	—4.93	—6.41
2	24.40	—4.10	—5.42
3	29.30	—3.41	—4.59
4	35.20	—2.84	—3.91
5	42.20	—2.37	—3.34
6	50.60	—1.98	—2.87
7	60.80	—1.64	—2.48
8	72.90	—1.37	—2.15
9	87.50	—1.14	—1.87
10	126.00	— .79	—1.45
Weighted harmonic average		—2.26	—3.35

With regard to the structure, it is clear that between two foodgrains (which both already are part of the diet) the substitutability will generally be greater than between either of them and, *e.g.*, vegetables. Thus, different levels of substitutability should be distinguished[11]. These levels can be assumed on the basis of observation of dietary habits. However, some more empirical evidence has been used also, in that price series for a number of food items have been compared. It appears that the price movements between wheat, other foodgrains and pulses are fairly closely related, but that rice stands rather apart from them. This is to be expected since the provincial averages per consumer, given in the QSCEC, hide the fact that in most parts of the province either wheat or rice is the staple foodgrain, so that little direct substitution takes place between them. Prices of sugar and *gur* move closely together. However, foodgrain and sugar prices show relatively little relation.

[11]The concept is close to that used by Barten [3]. However, Barten directly links the substitution elasticities, while in this paper the link is between the changes in marginal utility.

As regards the level of substitutability, an empirical approach is possible in the sense that an assumption can be made, the results obtained and a comparison made with empirical results obtained elsewhere. Thus, the maximum degree of substitutability was put at the apparently "reasonable" level of .4, and the results, compared with, *e.g.*, those of Barten [3] seemed to be of the right order of magnitude. At the present stage, not much more can be said, and the final outcomes will have to be tested against Pakistan practice. The x_{ik} matrix is presented in Table III[12].

TABLE III

ASSUMPTIONS FOR THE SUBSTITUTABILITY FACTOR x_{ik} FOR 13 FOOD ITEMS*

Commodity	1	2	3	4	5	6	7	8	9	10	11	12	13
1. Wheat	1.0	.2	.4	.2	.3	.1	.1				.1	.1	.1
2. Rice	.2	1.0	.2	.1	.1	.1	.1				.1	.1	.1
3. Other foodgrains	.4	.2	1.0	.2	.2	.1	.1				.1	.1	.1
4. Baked prods.	.2	.1	.2	1.0	.1	.1	.1				.1	.1	.1
5. Pulses	.3	.1	.2	.1	1.0	.1	.1			.2	.1	.1	.1
6. Milk	.1	.1	.1	.1	.1	1.0	.2		.2				
7. Milk prods.	.1	.1	.1	.1	.1	.2	1.0	.2	.2	.1			
8. Edible oil							.2	1.0					
9. Meat, fish						.2	.2		1.0				
10. Fruits, vegetables					.2		.1			1.0			
11. Sugar	.1	.1	.1	.1	.1						1.0	.4	.1
12. *Gur*	.1	.1	.1	.1	.1						.4	1.0	.1
13. Misc. food	.1	.1	.1	.1	.1						.1	.1	1.0

* $$x_{ik} = \frac{u'_{ik}}{\sqrt{u'_{ii}\, u'_{kk}}} = \frac{\frac{\partial^2 U}{\partial A_i \partial A_k}}{\sqrt{\frac{\partial^2 U}{\partial A_i 2} \frac{\partial^2 U}{\partial A_k 2}}}$$ (See Appendix subsection A-2).

[12]It is sometimes argued that in the rural subsistence sector little substitution is physically possible. However, recent research by Massell [18] seems to contradict this notion. Although calculations for lower substitutability coefficients have been made for the rural sector, these are, therefore, not presented in the following section.

V. RESULTS

V.1 The Income Elasticities

Table IV gives the income elasticities (306 of them). It should be emphasized that they are income and not expenditure elasticities. Because of the steep increase in savings, especially in the rural sector, expenditure elasticities would come out higher.

Looking at the main groups, the overall results appear to be reasonable. So, *e.g.*, the figure for the all important food group lies in the range that is usually found. Especially in the rural sector, the elasticity of demand for food has the tendency to go down with income.

The last three main groups show clear differences between income groups. As was pointed out before, however, the high elasticities for the "Miscellaneous" group are disturbing. Especially the increasing elasticity for services in the rural sector may, therefore, be more apparent than real.

The picture for the main groups naturally fails to bring out important features in the demand for individual commodities. Generally speaking, one would also expect the registration of the consumption of items individually enumerated to give more precise results. A detailed discussion of the results for each commodity is not given since the table already gives the basic information. A few items are important enough to be mentioned separately, however.

For wheat, the main staple item in the diet, which provincewide takes about 18 per cent of all expenditure, a decreasing elasticity with income is apparent. In urban areas, the elasticity is even sharply negative in the upper income groups. The total average elasticity agrees closely with Hufbauer's (which was based on the same data) but is at variance with what most others have found. The possible reason is that many *a priori* specifications of Engel functions of course do not allow for the possibility of a downturn at the upper end.

The somewhat puzzling outcomes for commodities 3 and 4 (other foodgrains and baked products, respectively) are probably the result of non-homogeneity within the commodity. So, *e.g.*, baked products include ready made *chapatis* which are consumed by the lower income groups as well as bakery products for which the reverse is true.

The figures for milk and milk products (groups 6 and 7) suggest that these are items through which the lower income groups try to diversify their diet.

For sugar and *gur* (groups 11 and 12) the widely divergent results between urban and rural areas probably accurately reflect the different consumption patterns.

TABLE IV

INCOME ELASTICITIES

Commodity code*	Urban elasticities for income groups				Rural elasticities for income groups				Overall weighted
	Lower	Middle	Higher	Weighted	Lower	Middle	Higher	Weighted	
I	.62	.69	.56	.63	.69	.57	.54	.63	.63
II	1.02	1.00	.99	1.00	.63	.72	.75	.68	.77
III	.80	.87	1.14	.96	.59	.78	1.10	.73	.82
IV	1.91	1.31	1.01	1.32	.85	1.15	1.14	1.02	1.15
V	5.05	2.37	1.16	1.90	2.94	1.76	1.23	1.96	1.95
1	.08	.08	—.48	—.01	.36	.16	.11	.27	.21
2	1.59	1.05	.32	.98	1.07	.90	.59	.93	.95
3	1.99	1.38	—.89	.95	.57	.69	.92	.66	.69
4	1.78	1.37	.62	1.18	—4.23	3.27	2.40	—2.31	.34
5	0	.07	.19	.06	.15	.21	.44	.19	.16
6	1.02	.95	.42	.80	1.74	.70	.31	1.10	1.02
7	1.53	1.22	.95	1.19	1.20	.93	.60	1.01	1.04
8	.09	.18	.49	.21	—.32	—.09	.52	—.19	.05
9	1.46	1.14	.92	1.14	1.33	1.21	1.05	1.23	1.19
10	.75	.82	1.24	.96	.33	.47	.76	.43	.64
11	1.37	.86	.55	.93	.14	.36	.75	.28	.51
12	—1.23	—.46	—.47	—.92	1.77	1.31	1.24	1.46	.73
13	0	2.49	.45	.84	.73	.76	.90	.76	.77
14	.33	.42	.58	.43	.69	.72	.77	.71	.59
15	.45	.58	.87	.62	.67	.66	.70	.67	.66
16	.93	.93	.85	.90	1.81	2.20	1.47	1.90	1.06
17	1.09	1.05	1.07	1.07	.65	.75	.79	.71	.82
18	.77	.80	.46	.69	.55	.60	.61	.58	.61
19	.36	.45	.71	.49	.32	.21	.29	.28	.34
20	1.08	1.08	1.21	1.14	.77	1.25	1.53	1.10	1.12
21	2.08	1.47	1.82	1.78	1.89	1.19	.72	1.38	1.49
22	1.12	.86	.57	.85	.45	.52	.62	.50	.63
23	2.26	.89	.65	1.22	1.03	1.39	1.31	1.24	1.23
24	1.04	1.02	1.34	1.17	.57	.98	1.05	.81	.93
25	5.26	2.07	1.17	1.93	2.80	1.40	.67	1.72	1.86
26	3.21	1.68	.80	1.56	2.28	1.81	1.32	1.96	1.71
27	3.35	3.79	1.56	2.03	4.58	3.17	1.91	2.83	2.51
28	2.09	1.72	1.07	1.45	2.31	1.42	1.12	1.66	1.60
29	6.14	2.48	1.18	1.97	3.00	1.79	1.23	1.99	1.98

*For commodity code, *see* Appendix C.

Housing seems to be one of the major categories through which the upper income groups show their affluence. In the urban areas where power is available, the expenditures on rent are complemented by higher elasticities for furniture and appliances, and for fuel and light by the upper income group.

The very high elasticities for the lower income groups in the last three service sectors (education, recreation, domestics) suggest an urgent demand for these services even among the poor. One should keep in mind, however, that these elasticities relate to an extremely small base. In order to keep a proper perspective, Table IV should in any case be studied in conjunction with Appendix Tables B-5 and B-6, which give the marginal expenditure coefficients.

V.2 The Price Elasticities

The direct price elasticities are a combination of substitution and income effects. Under want-independence, the cross-elasticities are the results of income effects only[13], but as soon as want-dependence is recognized, the cross-elasticities become conglomerates also. The two components were calculated separately, but to avoid a mass of detail only the main results are presented in the text tables. However, matrices of the estimated substitution effects between the want-dependent food items are given in Appendix Tables B-7 and B-8. Income effects can be easily calculated from formula (19) in Appendix A. Thus the interested reader can reconstruct all the components, if so desired.

For the five main groups, Table V gives the numbers. A few things stand out. In the first place, the order of magnitude of the own-price elasticities. They lie between $-.3$ and $-.9$, and very roughly one could say that they generally do not differ too much from .5.

In the second place, and as one would expect, the own-price elasticities are smaller in case 2 (with the higher Z) than in case 1. However, the difference is less than proportional to the difference in the Z, because the income effect partly compensates the substitution effect.

In the third place, all cross elasticities are fairly low except for the influence of a change in food prices on consumption elsewhere. This is mainly due to the large budget proportion for food. When $Z=Z_2$, the cross elasticity between food and the other groups is generally even higher than the own-price elasticity of those groups.

Table VI gives the own-price elasticities for the commodities, and the split-up oversubstitution and income effects. Obviously, when the price of only one commodity changes, the income effect will be smaller than in the case when prices for a whole group change (Table V). However, since the income

[13]As defined in this paper (differently from Slutsky). *See* Appendix A. It should be realized that the income effect incorporates an effect on savings, since the income elasticity, which is used to calculate the income effect, takes savings into account.

effect for all main groups except food was already small, the average for the goods in each of those groups comes close to the figure given in Table V. Within each group, the spread is, however, fairly wide. The highest is the price elasticity for domestic help.

Within the food group, the price elasticity would tend to come out fairly low for most commodities, if want-independence were assumed. However, under the particular want-dependence postulated in this paper, own-price elasticities jump up substantially. As again one would expect, the increase is largest in items with a small budget proportion and/or large substitutability factors.

In the case $Z=Z_1$, half the own-price elasticities in the want-dependent food group attain values over .75. Only wheat remains low at .2 or less. This basically confirms the results of an earlier study which arrived at an estimate of .3 as a maximum [7].

TABLE V

OWN AND CROSS-PRICE ELASTICITIES e_{ik}, 5 MAIN GROUPS

i \ k	I	II	III	IV	V
A. Z = — 2.26					
I. Food, drink	—.51	—.30	—.29	—.42	—.77
II. Clothing, footwear	—.04	—.41	—.04	—.06	—.11
III. Housing, *etc.*	—.05	—.05	—.49	—.08	—.13
IV. Services	—.03	—.03	—.03	—.61	—.09
V. Miscellaneous	—.01	—.01	—.01	—.02	—.87
B. Z= —3.35					
I. Food, drink	—.46	—.33	—.33	—.47	—.85
II. Clothing, footwear	—.04	—.29	—.05	—.07	—.13
III. Housing, *etc.*	—.06	—.07	—.35	—.10	—.17
IV. Services	—.04	—.05	—.05	—.45	—.13
V. Miscellaneous	—.02	—.03	—.03	—.04	—.65

TABLE VI

OWN-PRICE ELASTICITIES e_{ii}, AND INCOME COMPONENT V_{ii}, ALL COMMODITIES

	Items	e_{ii} under want-independence[a]		e_{ii} want-dependent goods		(v_{ii})[b]	
		$Z=Z_1$	$Z=Z_2$	$Z=Z_1$	$Z=Z_2$	$Z=Z_1$	$Z=Z_2$
1.	Wheat	—.10	—.09	—.19	—.16	—.05	—.06
2.	Rice	—.36	—.26	—.72	—.52	—.02	—.02
3.	Other foodgrains	—.26	—.19	—1.08	—.79	—.01	—.01
4.	Baked products	—.35	—.22	—2.46	—1.72	0	0
5.	Pulses	—.08	—.05	—.84	—.59	0	0
6.	Milk	—.38	—.29	—.63	—.47	—.04	—.05
7.	Milk products	—.46	—.34	—.99	—.70	—.04	—.05
8.	Edible oil	—.08	—.05	—.37	—.24	0	0
9.	Meat, fish	—.57	—.40	—.89	—.61	—.02	—.03
10.	Fruits, vegetables	—.39	—.25	—.50	—.32	—.01	—.01
11.	Sugar	—.24	—.16	—.50	—.35	—.01	—.01
12.	*Gur*	—.31	—.22	—1.04	—.72	0	0
13.	Other food	—.31	—.23	—.45	—.32	—.01	—.01
14.	Condiments, spices	—.27	—.18			—.01	—.01
15.	Tea	—.29	—.20			—.01	—.01
16.	Tobacco	—.58	—.38			—.01	—.01
17.	Clothing	—.43	—.30			—.03	—.04
18.	Footwear	—.25	—.18			—.01	—.01
19.	Fuel, light	—.17	—.12			—.02	—.02
20.	Rent	—.65	—.45			—.02	—.04
21.	Furniture	—.76	—.49			0	—.01
22.	Personal care	—.29	—.20			—.02	—.02
23.	Medical exp.	—.55	—.38			—.01	—.02
24.	Transport	—.52	—.34			—.01	—.01
25.	Education	—.93	—.62			0	—.01
26.	Recreation. reading	—.78	—.53			0	—.01
27.	Domestic	—1.46	—.95			+.01	0
28.	Gifts, *etc.*	—.70	—.48			0	—.01
29.	Miscellaneous	—.88	—.66			—.03	—.08

[a] This assumption is unrealistic for the first 13 items. Figures have only been given for purposes of comparison.

[b] The income component of the price elasticity is the same, whether goods are assumed to be want-independent or not (see Appendix Section A.3).

Finally, all cross elasticities (except for the trivial ones) for $Z=Z_1$ are given in Table VII. Many factors determine the pattern of these cross elasticities, especially between the want-dependent goods. One conclusion that stands out very clearly is that wheat is the commodity of which the price influences demand for other products most deeply. This is of course not surprising, given its large budget proportion and low price elasticity. However, for a few goods for which large x_{ik} with wheat have been assumed, the negative income effect is overcome by the positive substitution effect. Thus, the demand of other foodgrains and pulses will be positively affected by an increase in wheat price. On the other hand, there is still a slight negative net effect on rice. These results are in close conformity with the price experience quoted in subsection IV.2 above.

Other rows with important entries in Table VII are milk and milk products (Rows 6 and 7), where also both the substitution and income components are important. Especially, milk products seem to be substituted by a whole range of other products, again what one would reasonably expect.

Columns that have a large number of entries generally have a low budget proportion (4), a high income elasticity (29) or both (25-27).

The substitution parts of the cross elasticities found were compared to the results Barten [3] estimated empirically for the Netherlands. For commodities with roughly the same kind of interrelation (in terms of budget shares, substitutability, *etc.*), the coefficients are of generally the same order of magnitude. This gives some further indication that the results probably are "reasonable".

VI. CONCLUDING REMARKS

As indicated earlier (Section I), this study originated as a part of an attempt to build a policy model for West Pakistan that would include price and income relationships. The question thus needs to be asked whether the above results could be used for this purpose.

One serious drawback noted earlier is that expenditures were given per household income group. Each income group, therefore, includes the average expenditure pattern of households with fairly widely varying levels of per capita income. As long as the expenditure relations are linear, this should not influence the results too much. If the "real" relations are curved, however, chances are that the averaging implicit in the data will lead to an underestimate in the differences in economic behaviour between income groups.

Another serious drawback is the fact that the split-up of expenditure is incomplete, especially in the rural sector, as indicated by the great importance of the "All other miscellaneous" category. As is the case for the first reservation mentioned above, this drawback can only be (partially) removed by going back to the original observations.

TABLE VII

CROSS-PRICE ELASTICITIES e_{ik} $(i \neq k) \times 100$, ALL COMMODITIES ASSUMING $Z=Z_1$

i \ k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1		−9	23	50	24	−19	−15		−20	−11	−5	−8	−12	−10	−11	−14	−13	−10	−6	−16	−23	−10	−20	−13	−27	−26	−33	−26	−33
2			11	8					−5		3	8	3								−3				−3	−3	−4	−3	−4
3	3	6		38	7				−3			3															−3		−3
4			8				4					5	3																
5			6						−4	9		5													−3	−3	−3	−3	−3
6			4	42	10		5	−6	12	−5	−4	−9	−6	−3	−3	−4	−3	−3		−5	−6	−3	−6	−4	−8	−7	−11	−7	−9
7		6	12	72	8	6		52	22	11	−5	−11	−7		−3		−3			−4	−5		−4	−3	−5	−5	−9	−6	−8
8				−9			9		−5																−4	−3			
9		−4	−4	−20	−3	7	11	−7																	−3	−3	−3	−3	−3
10		−3	−4	−6	16	−4	4	−3	−3																−3	−3	−3	−3	−4
11				13	3	−3						46													−3	−3	−3		−3
12				9							15																		
13				18	4	−3	−3					4															−3		−3
15																													−3
17		−5	−4			6	−5		−5			−4	−4	−3		−4		−3		−4	−6	−3	−6	−3	−7	−7	−8	−7	−9
18																											−3		−3
19		−5	−3			−5	−5		−5				−3		−3	−4	−3	−3		−4	−6	−3	−5	−3	−8	−7	−9	−7	−8
20		−3				−3	−3		−3							−3					−3		−3		−5	−5		−4	−4
22		−3				−3	−3		−3							−3				−3	−4		−3		−5	−5	−5	−4	−5
29																											−3		

Note: For commodity code, see Appendix C.
All entries with an absolute value below .03 have been omitted.

However, the original survey data for most concrete commodities are probably reasonably accurate. The technique followed in this paper has tried to extract from these data as much information as possible which can be brought to bear on the immediate policy problems at hand. Although the method rests on many assumptions, the quantitative results (as far as they can be judged at this stage) do not seem to be in obvious conflict with reality. Therefore, the findings reported are probably the best presently available for the large study. Also the method that we developed may be of similar use elsewhere.

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Appendix A

RELATIONS USED TO DERIVE INCOME AND PRICE ELASTICITIES

Variables are verbally defined as they are introduced. A complete set of definitions is given (Subection A.5) at the end of this Appendix.

All incomes and expenditures are defined in per capita terms. Except for Subsection A.4, all relations refer to one particular income group.

A.1 Income Elasticities per Income Group

For each level of income Y, the savings relation derived in Section III of the main text specifies the absolute savings level S, the average savings rate s and the marginal savings rate $\dot{s}$.

Similarly, the Engel functions specify for each consumption item the absolute expenditure level A_i, the average budget proportion a_i and the marginal budget proportion, $\dot{a}_i$. These proportions relate to total expenditures A but, given the savings rates, they can be converted to proportions of Y:

$$\hat{a}_i = \frac{A_i}{Y} = \frac{A_i}{\frac{A}{(1-s)}} = (1-s)a_i \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$\dot{\hat{a}}_i = \frac{dA_i}{dY} = \frac{dA_i}{\frac{dA}{(1-s)}} = (1-\dot{s})\dot{a}_i \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

The income elasticity is defined as

$$E_i = \frac{dA_i}{dY} \cdot \frac{Y}{A_i} = \frac{\dot{\hat{a}}_i}{\hat{a}_i} = \frac{(1-\dot{s})\dot{a}_i}{(1-s)a_i} \quad \ldots\ldots\ldots\ldots \quad (3)$$

Since all items on the right-hand side of (3) are given, the income elasticity can be calculated.

A.2 Price and Cross Elasticities per Income Group

A.2.1 The decomposition of the price and cross elasticities: The price and cross elasticities e_{ik} are divided into a substitution part s_{ik} and an income part v_{ik}:

$$e_{ik} = s_{ik} + v_{ik} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Using p and q to denote prices and quantities, respectively, the substitution component is defined as

$$s_{ik} = \frac{dq_k}{dp_i} \cdot \frac{p_i}{q_k} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

under the condition that the marginal utility of money is kept constant.

Because of this last condition, the decomposition of the total price elasticity into income and price effects, as adopted in this paper, is difficult from the one introduced by Slutsky. Under the Slutsky definition, the indifference level is held constant, and this does not imply a constant marginal utility of money (income). However, changes in income influence the consumption pattern exclusively through a change in the marginal utility of money. Therefore, it is more efficient to include all such effects in the income part of the price elasticity, and to exclude them from the substitution elasticity.

A.2.2 Derivation of the substitution effects: If U is an indicator of the total utility of a "representative consumer", the marginal utility of income can be defined as

$$w = \frac{dU}{dy} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

and the marginal utility of good i as

$$u_i = \frac{dU}{dq_i} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

In equilibrium, the marginal utility of one rupee spent on any good should be equal to the marginal utility of income (or money):

$$\frac{u_i}{p_i} = w \text{ or } u_i = p_i w \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

The elasticity of marginal utility of good k with respect to consumption of good i can be defined as

$$z_{ik} = \frac{du_k}{dq_i} \cdot \frac{q_i}{u_k} = \frac{d^2U}{dq_i\, dq_k} \cdot \frac{q_i}{u_k}$$

$$= \frac{d^2U}{dA_i\, dA_k} \cdot \frac{A_i}{w} = u'_{ik} \frac{A_i}{w} \qquad \ldots\ldots\ldots\ldots\ldots \quad (9)$$

Consider the effect of an increase in p_i by 1per cent, leaving all other p and w unchanged. Using (8), this implies that in the new equilibrium, u_i will be 1 per cent higher, and that all other u_k ($k \neq i$) remain unchanged. On can

then write:

$$\left.\begin{aligned} \sum_j s_{ij} z_{jk} &= 0 \text{ for } k \neq i \\ \sum_j s_{ij} z_{ji} &= 1 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

If there are m sectors, the m relations (10) express the m s_{i1}, s_{i2}——s_{im} as functions of the z_{jk}. Assuming price increases in other sectors, the same relations can be developed for all other s_{jk}. Thus, all s_{ij} can be expressed in the z_{jk}. In matrix terms, the s_{ik} matrix is the inverse of the z_{ik} matrix.

The z_{ik} can be determined as follows:

Analogous with (9), define the elasticity of the marginal utility of money with respect to income as:

$$Z = \frac{dw}{dy} \cdot \frac{Y}{w} = \frac{d^2U}{dY^2} \cdot \frac{Y}{w} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

Consider the effect of an increase in Y by 1 per cent, leaving all p_i unchanged. According to (11) and (8), w and all u_i will then increase by Z per cent. One can then write:

$$\sum_i E_i z_{ik} = Z \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

These m relations determine the system if all z_{ik} can be expressed in the m diagonal elements z_{ii}. This can be done in a reasonably realistic manner by assuming relations between the second derivatives of the utility function:

$$u'_{ik} = \frac{d^2U}{dA_i\, dA_k} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

This derivative indicates the change in marginal utility of expenditure on k, caused by an increase of one rupee in expenditures on i. If two goods are perfect substitutes, u'_{ik} will be equal to u'_{ii} ; if they are not related, u'_{ik} will be 0.

Therefore, a substitutability factor x_{ik} ($-1 \leqslant x \leqslant 1$) is hereby introduced, such that $\hat{u}'_{ik} = 0$ if $x_{ik} = 0$, and $\hat{u}'_{ik} = \hat{u}_{ii}$ if $x_{ik} = 1$. Because $\hat{u}'_{ik} = u'_{ik}$ the link between u'_{ik} and $\hat{u}_{ii}$ has to be symmetric in i and k. Thus,

$$u'_{ik} = x_{ik}\sqrt{u'_{ii} + u'_{kk}} \rightarrow \tfrac{1}{2} x_{ik}(u'_{ii} + u'_{kk}) \quad \ldots\ldots\ldots\ldots \quad (14)$$

Substitution of (14) in (9) yields:

$$z_{ik} = \tfrac{1}{2}\frac{A_i}{w} x_{ik}(u'_{ii} + u'_{kk}) = \tfrac{1}{2} x_{ik}\left(z_{ii} + \frac{a_i}{a_k} + z_{kk}\right) \ldots\ldots \quad (15)$$

* Since u_{ik} and u_{ki} are second derivatives of a continuous function, symmetric in i and k.

Thus, the relations (15) and (12) express the unknown z_{ik} in x_{ik}, Z and E_i, which are all known. Once the z_{ik} have been derived, the relations (10) yields the s_{ik}.

A.2.3 Derivation of the income effects: Consider the effect of an increase in p_i by 1 per cent leaving all other p unchanged. This will tend to increase the total amount spent by $(\sum_j s_{ij} a_j + a_i)$ per cent. If, however, total income is also unchanged, these increases in expenditure decrease the amounts available for consumption in the same way as a decrease in income world. Thus, the income elasticities determine the division of this income effect, such that:

$$v_{ik} = -(\sum_j s_{ij} a_j + a_i) E_k \quad \ldots \quad (16)$$

Given this, the total price and cross elasticities are given by (4).

A.3 The Special Case of Want-independence

Under special conditions, the relations derived in this Appendix can be greatly simplified. These conditions are those of want-independence.

A commodity is defined to be want independent if any increase in its consumption does not influence the utility of any other goods. In other words, good i is want-independent if $x_{ik} = 0$ for all $k \neq i$.

This implies that all z_{ik} and $s_{ik} = 0$ for $k \neq i$.

In this case, relation (12) becomes

$$E_i z_{ii} = Z \text{ or } z_{ii} = \frac{Z}{E_i} \quad \ldots \quad (17)$$

and the relation (10) is simplified to:

$$s_{ii} z_{ii} = 1 \text{ or } s_{ii} = \frac{1}{z_{ii}} = \frac{E_i}{Z} \quad \ldots \quad (18)$$

Furthermore, (16) becomes

$$v_{ik} = -a_i E_k \left(1 + \frac{E_i}{Z}\right)^* \quad \ldots \quad (19)$$

which implies for (4)

$$e_{ik} = v_{ik} = -a_i E_k \left(1 + \frac{E_i}{Z}\right) \text{ for } k \neq i \quad (20)$$

and

$$e_{ii} = s_{ii} + v_{ii} = \frac{E_i}{Z} - a_i E_i \left(1 + \frac{E_i}{Z}\right)$$

*It can be proved that this particular simplified formula has general validity: it also applies when the goods are not want independent. In other words, the income part of the price elasticities is the same, whether goods are want-independent or not.

Given these simplifications, it is clear that the assumption of want independence is an attractive one.

A.4 Weighting of Elasticities over Income Groups

Composite elasticities can be derived by weighting over income groups. In the case of price and cross elasticities, such weighting will generally be indicated, since price changes for a certain commodity will normally be identical for all income groups. However, in the case of income elasticities, weighting over income groups only makes sense if all incomes go up in the same proportion. This will generally be an unwarranted assumption, and weighting will then lead to loss of information. Nevertheless, to limit the amount of coefficients to be presented, income elasticities have in this paper been weighted to arrive at coefficients for thru main income categories.

In weighting elasticities over income groups, the weights W_i^n have to be proportional to the amounts spent on good i by each income group w. Thus:

$$W_i^n = \frac{y^n \; A_i^n}{\sum\limits_n y^n \; A_i^n} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (21)$$

in which y^n gives the income distributions, such that $\sum\limits_n y = 1$

The weighted elasticities then are:

$$\bar{E}_i = \sum_n W_i^n \, E_i^n \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (22)$$

and

$$\bar{e}_{ik} = \sum_n W_k^n \, e_{ik}^n \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (23)$$

A.5 Symbols and Definitions

All variables are defined per income group, except as otherwise noted. All incomes and expenditures are in per capita terms.

A=total expenditure in Rs./month.

A_i=expenditure on good i in Rs./month.

$a_i = \dfrac{A_i}{A}$ = proportion of expenditure spent on goods i.

$\hat{a}_i = \dfrac{A_i}{y}$ = proportion of income spent on good i.

$\dot{a} = \dfrac{dA_i}{dA}$ = marginal proportion of expenditure spent on good i.

$\hat{a}_i = \dfrac{dA_i}{dY}$ = marginal proportion of income spent on good i.

$E_i = \dfrac{dq_i}{dY} \cdot \dfrac{Y}{q_i} = \dfrac{dA_i}{dy} \cdot \dfrac{Y}{A_i}$ = income elasticity of good i.

$\bar{E}_i = \sum_n W_i^n E_i^n$ = weighted income elasticity of good i.

$e_{ik} = \dfrac{dq_k}{dp_i} \cdot \dfrac{p_i}{q_k}$ = elasticity of good k with respect to price of i.

$\bar{e}_{ik} = \sum_n W_k^n e_{ik}^n$ = weighted elasticity of good k with respect to price of i.

p_i = price of good i.

q_i = quantity of good i.

S = total savings in Rs./month.

$s = \dfrac{S}{y}$ = proportion of income saved.

$s' = \dfrac{dS}{dY}$ = marginal proportion of income saved.

$S_{ik} = \dfrac{dq_k}{dq_i} \cdot \dfrac{p_i}{q_k}$ = substitution component of elasticity of good k with respect to price of i.

U = utility indicator for "representative consumer".

$u_i = \dfrac{dU}{dq_i}$ = marginal utility of good i.

$u'_{ik} = \dfrac{d^2U}{dA_i\, dA_k}$ = second derivative of utility with respect to amount spent on goods i and k.

$v_{ik} = e_{ik} - s_{ik}$ = income component of elasticity of good k with respect to price of i.

$$W_i^n = \frac{y^n A_i^n}{\sum_n y^n A_i^n} = \text{proportion of good i consumed by income group n.}$$

$$w = \frac{dU}{bY} = \text{marginal utility of money (income).}$$

$$x_{ik} = \frac{u'_{ik}}{\sqrt{u'_{ii}\ u'_k}} \rightleftharpoons \frac{u'_{ik}}{\frac{1}{2}(u_{ii}+u'_{kk})} = \text{substitutability factor between goods i and k.}$$

$Y = A+S$ = total income in Rs./month.

y^n = proportion of income earned by income group n.

$$Z = \frac{dw}{dY} \cdot \frac{Y}{w} = \text{elasticity of marginal utility of money with respect to income.}$$

$$z_i = \frac{d_{uk}}{dq_i} \cdot \frac{q_i}{u_k} = \text{elasticity of marginal utility of good k with respect to consumption of good i.}$$

TABLE B-1

RESULTS OF TESTS OF DIFFERENT SPECIFICATIONS OF EXPENDITURE RELATIONS (URBAN)

(Entries give R^2 of relation[a])

Commodity	Linear relations with[b]						Semi-log with[b]		Double-log with[b]	
	A	A^2	$A+A^2$	$A+A^2+A^3$	$A+A^2+H$	A^2+H	A	A+H	A	A+H
Wheat			(.538)							
Rice			.964		.982		.956	(.971)		.965
Baked prods.							.847			
Pulses	(.347)									(.491)
Milk	.947		.993						.969	
Milk prods.	.994			.999					.974	.990
Ed. oil							.713		.707	
Meat, fish	.993			.997					.957	.993
Fruits, veg.	.909*			.993					.954	
Cond. spices	.943			(.971)*					.940	
Sugar			.974*				.981			.949
Gur							.513			
Tea						.985	.897			
Tobacco							.844		.917	
Misc. food					(.596)*		(.297)	(.529)*		
Clothing	.989		(.992)*					.926	.980	
Footwear			.967*				.974			.976
Fuel, light		.980					.883	(.931)		.971
Rent							.822	(.881)		(.959)
Furniture			(.953)*	.986			.704			
Personal care				.987			.957			
Medical exp.							.740		.755	
Transport				(.972)			.861			.943
Education, recr., reading	.947						.947			
Domestic							.808		.843	
All other				(.987)*			.914		.912	.963

[a]Explanation of entries:
Plain entry: All coefficients significant at the 99-per-cent probability level
Entry between ( ) „ „ „ 95-per-cent „
Entry with*: Constant in equation has less than indicated probability level
No entry: At least one of the coefficients for the explanatory variables is insignificant at the 95-per-cent probability level.

[b]A=total expenditure
H=average household size

TABLE B-2

ESTIMATED INFLUENCE OF HOUSEHOLD SIZE ON PER CAPITA CONSUMPTION URBAN (average household size 5.9)

Income group in QSCEC	2	6	10
Av. household size	3.4	7.1	8.8
Average per capita expenditure (Rs./month)	24.04	36.04	69.74

Per capita expenditure → / Commodity ↓	Actual	Deviation caused by household size	Related to income	Actual	Deviation caused by household size	Related to income	Actual	Deviation caused by household size	Related to income
Milk products	.75	—.14	.89	1.68	.06	1.62	3.67	.26	3.41
Meat, fish	.87	—.24	1.11	2.24	.12	2.12	4.53	.48	4.05
Tobacco	.67	.06	.61	.91	—.04	.95	1.34	—.16	1.50
Fuel, light	1.90	.15	1.75	2.09	—.06	2.15	3.04	—.20	3.24
Rent	2.22	.36	1.86	3.10	—.21	3.31	5.81	—1.03	6.84
Transport	.45	—.19	.64	1.62	.13	1.49	2.43	.65	1.78
		0			0			0	

TABLE B-3

URBAN ENGEL FUNCTIONS

PER CAPITA EXPENDITURES IN PAISA (1/100 Rp) PER MONTH

Income group →	1	2	3	4	5	6	7	8	9	10
Total expenditure →	2076	2453	2903	3439	4070	4814	5662	6658	7773	10590
Expenditure on[a] ↓										
1	490	497	504	513	523	530	526	511	481	365
2	48	67	89	115	141	166	184	200	213	228
3	8	12	18	24	32	40	45	47	44	19
4	11	16	23	30	39	49	60	71	80	94
5	65	65	65	65	65	57	68	70	73	78
6	133	161	195	234	280	331	379	424	462	508
7	64	89	118	152	191	236	286	345	411	577
8	102	105	107	109	112	117	123	132	143	164
9	82	112	146	186	230	280	336	403	478	667
10	90	103	119	137	158	184	219	267	331	551
11	51	67	86	104	122	140	159	179	198	238
12	51	44	35	30	29	27	26	24	21	18
13	12	12	12	13	20	37	65	90	106	98
14	53	57	60	65	69	75	82	90	100	125
15	38	42	45	50	55	61	70	80	93	131
16	57	68	80	95	113	134	158	185	217	296
17	141	174	212	258	311	377	459	561	684	1024
18	44	50	58	67	78	90	101	112	121	140
19	172	183	196	211	229	250	275	307	346	460
20	180	222	271	328	398	486	605	766	969	1539
21	6	10	14	20	26	33	44	60	87	190
22	95	119	146	177	208	239	265	292	322	398
23	25	57	86	112	131	147	162	180	201	258
24	49	60	73	87	105	126	154	192	243	420
25	0	18	48	85	128	178	236	304	379	571
26	5	16	28	43	61	80	98	117	137	177
27	0	02	04	10	25	49	79	118	162	279
28	4	10	14	19	26	35	49	65	81	112
29	0	15	51	100	165	250	350	466	590	864

[a]For commodity code, see Appendix C.

TABLE B-4

RURAL ENGEL FUNCTIONS

PER CAPITA EXPENDITURE IN PAISA (1/100 Rp) PER MONTH

Income group	1	2	3	4	5	6	7	8	9	10
Total expenditure	1992	2320	2731	3189	3716	4317	5023	5829	6771	9276
Expenditure on[a] ↓										
1	548	588	628	659	681	696	710	722	737	774
2	57	69	85	102	122	141	163	182	200	230
3	50	55	62	69	78	90	106	125	148	211
4	9	05	02	01	01	03	06	10	15	30
5	53	54	56	58	60	63	66	71	77	97
6	111	160	213	261	302	331	355	375	394	430
7	138	173	217	266	319	371	426	474	523	629
8	26	25	23	22	21	21	22	24	26	37
9	42	54	69	88	110	137	168	208	245	355
10	67	71	76	81	88	98	111	125	146	199
11	56	57	59	61	65	71	79	91	105	146
12	9	13	18	23	30	37	47	59	74	114
13	61	69	80	92	106	122	142	166	196	283
14	20	23	26	29	34	39	44	51	58	78
15	47	53	60	68	77	87	98	111	126	166
16	2	03	04	06	09	14	19	26	32	50
17	166	186	213	245	282	324	374	430	496	672
18	45	50	55	62	69	77	86	96	107	137
19	140	148	158	166	173	179	186	194	205	242
20	91	103	121	145	180	234	306	409	544	959
21	11	16	23	30	38	46	54	62	70	84
22	80	86	95	104	114	126	140	156	175	225
23	31	37	45	58	74	97	125	160	202	317
24	49	53	61	70	84	102	125	151	183	265
25	5	10	17	24	32	39	46	58	59	68
26	2	04	06	10	14	19	23	31	39	65
27	1	03	07	15	28	47	73	107	149	279
28	8	13	20	27	35	45	56	69	84	125
29	67	138	232	347	490	661	866	1095	1356	2009

[a]For commodity code, see Appendix C.

TABLE B-5

URBAN MARGINAL EXPENDITURE COEFFICIENTS

Commodity[a] ↓ / Income group →	1	2	3	4	5	6	7	8	9	10
1	.016	.016	.016	.016	.012	.006	—.020	—.030	—.030	—.045
2	.050	.050	.050	.048	.036	.028	.020	.014	.010	.004
3	.012	.012	.012	.012	.012	.010	.005	.001	—.004	—.010
4	.014	.014	.014	.014	.014	.014	.012	.010	.008	.004
5	0	0	0	0	002	.002	.002	.002	.002	.002
6	.074	.074	.074	.075	.070	.066	.054	.042	.030	.012
7	.064	.064	.064	.063	.061	.060	.059	.059	.059	.059
8	.004	.004	.004	.004	.006	.007	.008	.009	.010	.012
9	.080	.076	.076	.073	.067	.067	.067	.067	.067	.067
10	.034	.034	.034	.034	.034	.036	.042	.061	.061	.084
11	.052	.042	.042	.032	.026	.025	.024	.019	.017	.013
12	—.030	—.020	—.020	—.006	—.002	—.002	—.002	—.002	—.002	—.001
13	0	0	0	.002	.019	.027	.033	.023	.011	—.012
14	.008	.008	.008	.008	.008	.008	.008	.008	.009	.009
15	.008	.008	.008	.008	.009	.009	.010	.011	.012	.014
16	.028	.028	.028	.028	.028	.028	.028	.028	.028	.028
17	.085	.085	.085	.085	.087	.090	.098	.015	.112	.014
18	.017	.017	.017	.017	.017	.016	.012	.010	.008	.006
19	.028	.028	.028	.028	.028	.028	.030	.033	.036	.042
20	.108	.108	.108	.108	.114	.124	.145	.170	.188	.206
21	.010	.010	.010	.010	.010	.010	.013	.019	.026	.040
22	.064	.060	.060	.056	.044	.038	.028	.027	.027	.027
23	.080	.064	.064	.046	.022	.018	.018	.018	.019	.012
24	.028	.028	.028	.028	.028	.030	.034	.040	.048	.068
25	.068	.068	.068	.068	.068	.068	.068	.068	.068	.068
26	.028	.028	.028	.028	.026	.024	.021	.019	.017	.013
27	.004	.004	.004	.014	.030	.034	.037	.039	.040	.042
28	.010	.010	.010	.010	.012	.014	.016	.016	.014	.010
29	.056	.080	.080	.092	.112	.116	.120	.115	.109	.093

[a]For commodity code, see Appendix C.

TABLE B-6

RURAL MARGINAL EXPENDITURE COEFFICIENTS

Commodity [a] ↓ / Income group →	1	2	3	4	5	6	7	8	9	10
1	.130	.120	.080	.050	.030	.020	.017	.015	.015	.015
2	.038	.038	.038	.038	.036	.030	.027	.022	.018	.010
3	.016	.016	.016	.016	.018	.022	.023	.024	.025	.025
4	—.021	—.012	—.004	.000	.002	.004	.005	.005	.005	.006
5	.004	.004	.004	.004	.004	.005	.006	.006	.007	.008
6	.175	.146	.116	.088	.064	.034	.029	.023	.019	.013
7	.109	.108	.106	.014	.098	.077	.068	.057	.050	.040
8	—.004	—.004	—.004	—.002	.000	.001	.002	.002	.003	.005
9	.034	.036	.040	.042	.044	.044	.044	.044	.044	.044
10	.012	.012	.012	.012	.014	.018	.019	.020	.021	.021
11	.004	.004	.004	.006	.008	.012	.013	.014	.015	.017
12	.012	.012	.012	.012	.012	.014	.015	.015	.016	.016
13	.026	.026	.026	.026	.026	.028	.029	.013	.032	.036
14	.008	.008	.008	.008	.008	.008	.008	.008	.008	.008
15	.018	.018	.018	.017	.017	.016	.016	.016	.016	.016
16	.002	.002	.004	.005	.007	.008	.008	.008	.007	.007
17	.055	.060	.070	.070	.070	.070	.070	.070	.070	.070
18	.014	.014	.014	.014	.014	.014	.012	.012	.012	.012
19	.028	.026	.020	.014	.012	.010	.010	.011	.012	.016
20	.034	.038	.048	.060	.074	.102	.115	.134	.147	.172
21	.016	.016	.016	.016	.016	.014	.012	.011	.008	.005
22	.020	.020	.020	.020	.020	.020	.020	.020	.020	.020
23	.016	.018	.024	.030	.034	.040	.042	.044	.045	.046
24	.012	.014	.020	.024	.028	.023	.033	.033	.033	.033
25	.013	.016	.016	.016	.014	.010	.009	.007	.006	.003
26	.005	.006	.006	.008	.008	.008	.008	.008	.009	.011
27	.004	.006	.014	.022	.028	.036	.039	.044	.046	.054
28	.016	.016	.016	.016	.016	.061	.016	.016	.016	.016
29	.204	.216	.240	.264	.280	.290	.286	.282	.275	.255

[a] For commodity code, see Appendix C.

TABLE B-7

SUBSTITUTION COMPONENT s_{ik} OF ELASTICITY OF DEMAND FOR k WITH RESPECT TO PRICE OF i

Want-dependent commodities, $Z = Z_1$

i[a] \ k[a]	1	2	3	4	5	6	7	8	9	10	11	12	13
1	—.14	.07	.36	.46	.26	0	.03	—.02	—.01	—.02	0.2	.04	.01
2	.01	—.70	.12	.08	.02	.03	.05	—.02	—.03	—.01	.04	.09	.05
3	.04	.07	—1.07	.36	.07	.02	.03	—.02	—.02	—.01	.02	.04	.02
4	.01	.01	.08	—2.46	0	.02	—.04	—.02	—.02	—.01	.02	.05	.02
5	.02	.01	.07	.01	—.84	.03	.02	—.01	—.02	.10	.02	.06	.03
6	0	.06	.06	.44	.11	—.59	.10	—.06	.17	—.03	—.02	—.04	—.02
7	.02	.10	.12	.71	.09	.11	—.95	.52	.26	.13	—.04	—.07	—.03
8	0	—.01	—.01	—.08	—.01	—.01	.10	—.37	—.03	—.01	0	.01	0
9	0	—.02	—.03	—.19	—.03	.08	.13	—.07	—.86	—.01	.01	.02	.01
10	0	—.01	—.02	—.06	.16	—.01	.05	—.03	—.01	—.49	0	—.01	0
11	0	.03	.03	.13	.04	0	—.01	.01	.01	0	—.49	.48	.02
12	0	.02	.02	.09	.03	—.01	—.01	0	.01	0	.16	—1.04	.02
13	0	.04	.03	.17	.04	—.01	—.01	0	.01	0	.02	.05	—.44

a) For commodity, code *see* Appendix C.

TABLE B-8

SUBSTITUTION COMPONENT s_{ik} OF ELASTICITY OF DEMAND FOR k WITH RESPECT TO PRICE OF i

Want-dependent commodities $Z = Z_2$

i[a] \ k[a]	1	2	3	4	5	6	7	8	9	10	11	12	13
1	—.11	.05	.27	.35	.19	0	.02	—.01	—.01	—.02	.02	.03	.02
2	.01	—.50	.09	.05	.01	.02	.03	—.02	—.02	—.01	.03	.06	.03
3	.02	.05	—.78	.25	.05	.01	.02	—.01	—.01	—.01	.01	.03	.02
4	.01	.01	.06	—1.72	0	.02	.02	—.02	—.02	0	.01	.04	.02
5	.02	0	.05	0	—.58	.02	.01	0	—.01	.06	.02	.04	.02
6	0	.05	.05	.31	.08	—.42	.08	—.04	.12	—.02	—.01	—.03	—.01
7	0	.07	.08	.49	.06	.08	—.66	.36	.18	.09	—.02	—.05	—.02
8	0	—.01	—.01	—.05	—.01	—.01	.07	—.24	—.02	—.01	0	.01	0
9	0	—.02	—.03	—.13	—.02	.06	.09	—.05	—.59	—.01	0	.01	.01
10	0	0	—.02	—.04	.11	—.01	.04	—.02	—.01	—.31	0	0	0
11	0	.02	.01	.09	.03	0	0	0	0	0	—.34	.32	.01
12	0	.02	.01	.06	.02	0	0	0	0	0	.11	—.72	.01
13	0	.03	.02	.11	.03	—.01	—.01	0	.01	0	.02	.03	—.30

a) For commodity code, *see* Appendix C.

Appendix C

COMMODITY CODE

I. *Food, Drinks, Tobacco*

1. Wheat
2. Rice
3. All other foodgrains
4. Bakery products
5. Pulses
6. Milk
7. *Ghee*, butter and other milk products
8. Edible fats and oils
9. Meat, fish, poultry
10. Fruits, vegetables
11. Sugar (crystal)
12. Raw sugar and other sugarcane products (*gur*)
13. All other food and drinks, *n.e.s.*
14. Condiments, spices
15. Tea
16. Tobacco and chewing products

II. *Clothing, Footwear*

17. Clothing, bedding
18. Footwear

III. *Housing*

19. Fuel and lighting
20. House rent
21. Furniture and accessories

IV. *Services*

22. Personal care
23. Medical expenditure
24. Transport and travel
25. Education
26. Recreation, reading
27. Domestic help

V. *Gifts and Miscellaneous*

28. Gifts, charity
29. All other expenditure, *n.e.s.*