Book Reviews

Mathematical Economics by Kelvin Lancaster. Macmillan, New York, 1968.

The burgeoning of abstract economic analysis since about 1950 makes the need for well-conceived consolidations and codifications at the textbook level peculiarly important. The task is a challenging one, demanding the attainment of a compromise between the "mathematics for economists" catalogues of techniques and the highly specialized and formalized "theorem-proof" sequences of the high-theory journals. It requires that skilful blend of the rigorous and the heuristic, the multidimensional and the diagrammatic, the logical and the intuitive, found in the teacher-born. Lancaster has succeeded admirably in finding the optimal mixture.

One problem in writing such a book is the quite difficult if preliminary one of ogranizing the diverse material of an exploding field into sensible categories of presentation. The reviewer is not sure that Lancaster's are what he would choose: optimization theory, static economic models, and dynamic economic models. Such subsets are not disjoint, and their adoption separates some cognate fields. But, presumably, so would most if not all other convenient classifications be deficient in these respects, and it is not his intention to fault the author on his standard organizing choice.

It is surely true that there is emerging in applied mathematics, operations research, systems analysis, and economic theory — static and dynamic — a universalistic "theory of optimization", and it must be given pride of place as the predominant concern of economic theory. Lancaster devotes more than a third of his presentation of economic analysis proper to this theory, and does a fine job of presenting the material. One signal success of this portion of the book is the unified presentation of the basic tools of optimization. The author indicates that Kuhn-Tucker theory should be viewed as the most general "optimization-with-constraints" theory available with linear programming an easily derived subcase from the conditions of the general theory on the one hand, and Lagrangean techniques for interior maxima or minima not bounded away from negative values another special case.

Although this viewpoint is made explicit in the book, Lancaster did not choose to adopt it as an organizing principle for his treatment of optimization. Have we not now arrived at the point in microeconomic theory when we can start the student with Kuhn-Tucker analysis, lead into nonlinear programming as a general case, and move antihistorically but logically into classic calculus techniques on the one hand and linear programming on the other as analyses of simplification? Such an organization would have been much more consonant with Lancaster's quite consistent presentation of these topics in the unified vein; it is unfortunate that he did not opt to escape the confines of the conventional presentation sequence.

Some other minor choices of terminology or pedagogical technique may also be challenged. Why refer to a "continuous" closed set (p. 30) ? Why use the term "concave-contoured" for "quasi-concave", when the latter is now standard and the former will confuse the student when he moves to the literature ? Similarly, one can question the mixing of heuristic geometrically descriptive concepts with rigorous mathematical terms for functions in which Lancaster indulges when he speaks of transformation functions that are "convex outward". By convex outward he means concave, so that when on p. 134 he deals with the "sufficiency" conditions on the Hessian for such a function and states their negative definiteness, the student may well be puzzled. Why not an early definition of concave and convex functions and the construction of a consistent terminology on its basis?

Misprints in the book are very few: it shows a meticulous care by the author. But the inevitable few occur that might trouble the student in this part: on p. 24 it is stated that the typical linear programming problem has more constraints than variables; on p. 27 the origin is not identified as an extreme point of the linear programming simplex; on p. 36 the definition of the rank of a matrix has two errors; and on p. 62 the value of a function is miscalculated.

Part II of the book is a rather short treatment of static economic models: the Leontief closed and open models, activity analysis of production, neoclassical demand and production theory, and general equilibrium. The material is well, if hurriedly, presented. At two points the reviewer felt uncomfortable with the presentation because of an untypical conceptual fuzziness. First, on p. 139, the meaning of equilibrium is unclear, and second, Lancaster seems to feel on p. 141 that Walras' Law is unnecessary because the sum of the values of excess demands in equilibrium must be zero by the definition of market equilibrium and, he asserts, it is never necessary to solve for disequilibrium levels of excess demands. Why this militates against the need or usefulness of Walras' Law in general equilibrium — for example, in monetary theory — is simply not sufficiently amplified.

In Part III dynamic models are treated, with attention paid to a dynamic Leontief model, von Neumann models, efficient growth, the Turnpike theorem, and stability. It is nicely done — as compact and neatly unified a treatment as can be found in the literature today. The student who gains his baptism in optimal growth models from Lancaster's exposition will be a fortunate one.

The author has segregated the purely mathematical basis for the book from the more purely economic portions, relegating the former to a series of eleven "mathematical reviews" in a 170-page Part IV of his book. I believe that this experiment succeeds admirably. The author's concise presentation of the basic propositions with motivations or proofs, his appeals to intuition, his avoidance of the forbidding theorem-proof format without a great sacrifice in rigour, and his tying of the reviews to each chapter of the three economic analytical parts, deserve high praise. It is most carefully done, the reviewer noting only two troublesome slips. On p. 221, there is a confusing discussion that seems to imply that compact sets need not own their limit points. And on p. 339, because his prior assumptions for a Solow-Samuelson "sausage grinder" model (but which is not that model — a point not made sufficiently clear in the presentation) include that of nonnegative monotonicity, Lancaster's presentation of Morishima's extension of indecomposability is misleading. When the goods of a system are partitioned into two subsets, and the inputs of one such subset are increased, indecomposability of the system requires that at least one output in the other subset *change*. Lancaster's implies that such changes must be positive, when in fact they can be negative and meet the condition of indecomposability so that he has implicitly combined the concept with that of monotonicity of a system.

After reading such a concise, well-structured and beatifully executed codification of mathematical economics, as this one, the reader is led to ponder a bit the state of the core subjects in the field. The reviewer feels it is time to swing a bit away from the notion that the application of that difficult-to-delineate set of techniques that includes topology, activity analysis and programming, has revolutionized neoclassical economic theory to the extent of substantial supersession, by permitting important escapes from the calculus and continuous function theory. Admittedly, these techniques do permit an alternative presentation of consumer and production theory that possesses its own inherent sparkle; they do free some of our comparative statics proofs from the localness inhering in the calculus; and they give us tremendous leverage in such recondite areas as existence and uniqueness proofs. For these purposes they constitute an important methodological step forward, and are to be welcome.

But the danger — especially for the new math-oriented student — is that the enthusiasm of instructors and textbooks may blind to the fact that when we face the frontier problems of the field today we bump right back into the need to assume continuous, differentiable functions, and find ourselves back in the province of the old mathematics. Stability of systems, control theory, Kuhn-Tucker conditions — all gain much from the topologically oriented math; but they all must fall back upon the dreary old calculus in order to progress. Let us, therefore, encourage the student to master these new techniques alongside the old, and not seek to demonstrate our modernity and flexibility of learning sets by deprecating a solid foundation in the "old" math. Let us not, in short, delay the inevitable day of realization that the totally revolutionary — Keynes, input-output, game theory, linear programming, activity analysis — is seen to be substantial, measurable advance, but advance that integrates with older techniques and does not displace them. A Burkean continuity is not foreign to progress in the sciences — it is an historical theorem rarely disproved.

It is in this sector that the reviewer's one reservation about this fine book must be registered. Lancaster reveals a bit of the Grand Mission complex about the newer techniques; but the fact of the matter is that he does present the older techniques as well, if in the view of the reviewer he oversells the former. That said, however, this book is a most useful textbook for the field and should have a long life. It is recommended with enthusiasm.

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