The Evaluation of National Income in an Imperfect Economy

by

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In this paper, I shall discuss how national income should be measured in an 'imperfect' economy, where feasible policy instruments such as taxes, tariffs, quotas, and quantitative controls do not operate in a lumpsum manner, and may be far from their optimum level. In particular, I want to examine the meaning of national income, and its parts, when they are measured in terms of 'world prices'; or, more precisely and more generally, in terms of the accounting prices that might be computed as a guide to particular production decisions, in the public sector and elsewhere. I have found this set of questions confusing, and it may be that others do too, and will welcome an attempt at clarification.

I

We would like to be able to use national income statistics for a variety of purposes. Ideally, we should arrange that changes in aggregate national income reflect changes in social welfare; that changes in the income generated in each sector reflect changes in its contribution to welfare; and that changes in the income of a group, such as wage-earners or agriculturalists, reflect changes in the welfare of that group. Naturally, none of these ends can be achieved without first specifying a standard of group and social welfare, and people disagree about such standards. But it is worth emphasizing that changes in national income as usually measured need not, and usually do not, reflect changes in welfare. This is not to say that better measures of national income and its components cannot be devised. Certain general principles which I take to be of considerable importance can be stated: there is a standard theory of how national income should be measured at any rate for a 'perfect' economy[1]. I

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[1]Among the more important papers are: [2 ; 4 ; 6]. Professor Samuelson's paper [6] gives a definitive account of the theory, but not in the form sketched here.

shall begin by outlining this theory. By a 'perfect' economy, I shall mean an economy which is normally in competitive equilibrium, and in which property is so distributed that no conceivable transfers of property would increase welfare.

It is assumed that welfare is assessed in a way that respects individual preferences so that a move to a position preferred by one individual, while others remain in the same position, implies an increase in welfare. Then we can write social welfare as a function of the 'utilities' of individual.

$$W = W(u^1, u^2, \ldots, u^H) \tag{1}$$

where u^i is the utility of the i-*th* individual. The consumption of the i-*th* individual is denoted by $x^i = (x^i_1, \ldots, x^i_n)$, so that

$$u^i = u^i(x^i) = u^i(x^i_1, \ldots, x^i_n) \tag{2}$$

We consider what happens to welfare as the consumption bundles of the different individuals change. It is convenient to imagine that consumption is changing continuously through time. Then the utility of an individual is changing at a rate

$$\frac{d}{dt} u = \sum_{j=1}^{n} u_j \dot{x}_j \tag{3}$$

where u_j is the derivative of u with respect to x_j, the marginal utility of consuming the j-*th* commodity, and $\dot{x}_j$ is the rate of change of consumption of the j-*th* commodity. The assumption of competitive equilibrium means that marginal utilities are proportional to prices. Therefore

$$\frac{d}{dt} u = m \sum q_j \dot{x}_j \tag{4}$$

where $q = (q_1, \ldots, q_n)$ are the prices, and the factor of proportionality, m, is the 'marginal utility of income', and is positive. So, for the individual, it is natural to measure the change in utility by $\sum q_j \dot{x}_j$; that is, by the change in $\sum q_j x_j$ at constant prices.

We can now deduce the change in social welfare:

$$\frac{d}{dt} W = \sum_{i=1}^{H} W_i \frac{d}{dt} u^i \tag{5}$$

where W_i is the derivative of W with respect to u^i. From (4) and (5) we obtain:

$$\begin{aligned} \frac{d}{dt} W &= \sum_i W_i m_i \sum_j q_j \dot{x}^i_j \\ &= \sum_j q_j \sum_i W_i m_i \dot{x}^i_j \end{aligned} \tag{6}$$

The assumption that property is perfectly distributed implies that the marginal contribution of each man's income to social welfare is the same. That is, $W_1m_1 = W_2m_2 = \ldots = W_Hm_H = M$, say. (I shall call W_im_i the 'marginal welfare of the i-*th* individual's income'.) Then

$$\frac{d}{dt}W = M\Sigma_j q_j \dot{\Sigma_i x_j^i} \qquad (7)$$

Since $\Sigma_i x_j^i = x_j$ is the total supply of the j-*th* commodity, (7) demonstrates that changes in $\Sigma q_j x_j$ at constant prices may be taken to reflect changes in social welfare[2].

I incline to believe that the assumption of perfect property distribution is, quantitatively, the weakest link in the standard argument, which I have just outlined. When the measure of income is restricted to particular income groups in the economy, who might be supposed to have sufficiently similar W_im_i, it seems quite sensible to measure the change in *group* welfare by the change in group income, measured at constant prices. But, from the point of view of the theory, actual figures of *national* income do not seem to have much significance.

II

It is universally recognized that taxes or subsidies may be imposed on certain commodities where the individual's free choice is inconsistent with the maximization of social welfare, and that it may then be right to use the price, net of tax, to evaluate national income. But I am not aware of a published proof of this latter proposition. In any case, one would want to regard most commodity taxes, income taxes, and the like, as interference with the 'free market' required to finance public expenditure, and intended to improve the distribution of income. These taxes exist because desirable lumpsum taxes are impossible (or at least inordinately expensive). Not only are actual taxes far from being lumpsum transfers as required if competitive equilibrium is to be optimum: they are not chosen to maximize long-run social welfare, although welfare considerations undoubtedly influence them.

In such an economy, consumers purchase goods and services at market prices. At first sight, it may still seem reasonable to evaluate national income in terms of market prices, so long as we are primarily interested in assessing changes in real consumption. For any particular individual who takes reasonable care in allocating his expenditure, it is approximately true that he would be better off in an alternative (very similar) situation if and only if his consumption, measured at constant market prices, were higher in that situation. Thus, it is

[2]When the change of welfare during some interval of time is to be measured, it is, of course, desirable to construct an index of divisia type in which changes from each year to the next are weighted by the prices currently ruling, rather than the prices that ruled in some base year.

tempting to assert that an increase in national income (at constant market prices) means that, in some sense, people are on average better off. But this is quite misleading. We could deduce from such an increase that *someone* is better off; but that is a conclusion as weak as to be almost totally without interest. We *cannot* deduce that it is feasible to make *everyone* better off; for there may be no system of feasible taxes and subsidies that will allocate the available production as to increase everyone's utility[3]. The formal argument stated in Section I made it clear that national income at market prices is a satisfactory measure of increased welfare only if a small change in the income of any individual has the same effect on total welfare as an equal change in the income of anyone else: that is, if marginal welfares are equal.

That there are nevertheless 'prices' in terms of which it would be sensible to measure national income may be made plausible as follows. Granted a definition of social welfare, a small increase in the availability of one commodity should make possible an increase in social welfare, provided suitable tax changes are made. If we assume that before and after the change, tax and other policies are such as to bring about the maximum feasible welfare, the change in social welfare arising from an increase in the availability of a commodity by one unit may be taken to be its 'accounting price', measuring its true worth to the economy[4]. If one knew the accounting price that would be implied in the optimum feasible state of the economy, one would want production to maximize profits in terms of these accounting prices. For an increase in profits in one line of production would imply an increase in national income[5], and therefore, since for the moment everything is measured in terms of accounting prices, an increase in social welfare.

This simple argument suggests that suitable prices for evaluating national income can, in principle, be defined, and shows that the accounting prices so defined are the prices one would want for decentralizing production decisions. But two serious worries cry out for comment. First, the definition of the accounting prices was made on the assumption that social welfare was being maximized. Second, it is far from clear how such accounting prices could possibly be estimated. Nevertheless, it has been clearly demonstrated that

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[3]*Cf.* [6, pp. 18-19] Samuelson uses the term 'feasible' to describe institutionally possible arrangements of the economy.

[4]The argument is not watertight, for there may be no choice of taxes that will create an additional demand for the specified commodity, while leaving other demands unchanged; although welfare could be increased if at the same time the availability of some other commodity was slightly reduced. A rigorous proof of the existence of accounting prices (or ideal 'producer prices') under general conditions is given in [1].

[5]Given factor inputs. The argument is presented in a form that appears to assume that factor supplies are given, but this is merely an expository convenience. If there is a change in, say, the labour services supplied by households, the value of this change at constant prices should be subtracted from the value of the change in production in order to estimate the change in social welfare.

national income *in terms of market prices* has no special claim on the economist's attention.

I shall deal with the two worries in reverse order. First, let it be assumed that the government is indeed making the most of the policy instruments available to it. It is readily seen that, in some cases, the accounting prices may be quite obvious. The most important case is that in which all commodities can be traded with foreign countries at fixed prices, the same whether exported or imported. Then the trade prices, or 'world prices', must be the accounting prices. Commodity 1 can be substituted for commodity 2 in the ratio $p_1 : p_2$ (where $p_1, p_2, \ldots$ are the world prices). Therefore, the policies of government must have continued this substitution up to a point where the marginal gains to welfare from the two commodities are in the same ratio $p_1 : p_2$.

The situation may be portrayed simply in a diagram (Figure 1). Two commodities are produced. PP is the production frontier of the economy (in the absence of trade). The curves W_1, W_2, W_3 are equal-welfare contours, connecting production plans that give rise, with optimum government policies, to equal welfare. It will be noted that these contours will not in general be convex; but they do not, in general, intersect one another. On the reasonable assumption that more of either commodity makes possible an increase in welfare, it can be seen that X is the optimum point in the absence of trade. The line TT has slope $-p_1/p_2$: it represents the additional opportunities provided by trade. Then Y is the optimum point once trade is possible. The tangent to the welfare contour at Y gives the accounting prices, by our earlier argument.

It will be seen that the above argument rests on the simple observation that the welfare contours (or community indifference curves) of the usual analysis can perfectly well be replaced by feasible welfare contours to take account of the impossibility of the lumpsum transfers assumed in that analysis. A rigorous justification of this procedure is rather troublesome, however [1].

It is never true that all commodities can be traded at constant terms of trade. Some nontraded commodities[6] may be produced under constant returns to scale out of traded commodities, however. Indeed, it is known from the non-substitution theorem that, so long as there is only one nonproduced input (labour), constant returns to scale everywhere, and no capital prices can, in general, be determined uniquely. The presence of capital equipment causes trouble: in order to include capital costs in the costs of nontraded goods (as one must do in the case of nontraded goods) one must know the (accounting)

[6]And commodities sold to a market whose demand is relatively inelastic, for in these cases a country bent upon maximizing its own welfare even at the expense of others, will have to know the accounting price (or cost of production in terms of world prices) in order to know how much to export.

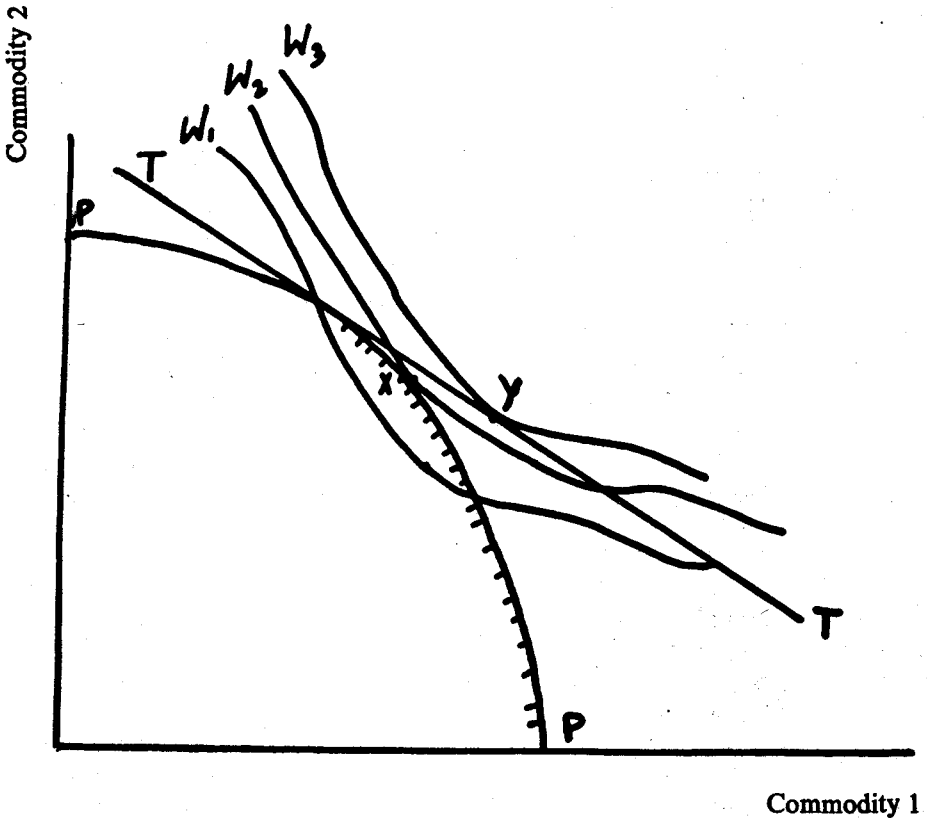

Figure 1

rate of interest, and the rate at which the (accounting) prices of capital goods are changing. If the country had access to a perfect capital market, the accounting rate of interest could be determined from the world price. But otherwise, it will depend on the particular social welfare function. Sometimes, accounting prices may not be very sensitive to the particular rate of interest chosen, of course. Attempts have been made to estimate parts of national income in accounting prices[7]: the task does not seem impossibly difficult although alternative prices, depending on alternative welfare specifications, may have to be calculated.

III

I shall now outline the more formal derivation of the accounting prices, and outline the relation between market prices and accounting prices in an optimally controlled economy of the type we have been considering. It must be assumed, and it is not unreasonable to do so, that the government can, by means of commodity taxes and subsidies, arrange for market prices to be anything it pleases. As before, market prices are denoted by $q = (q_1, \ldots, q_n)$. I denote by $V(q) = V(q_1, \ldots, q_n)$ the welfare that arises from the optimum use of other government policies, given the market prices. When these policies are followed, the aggregate demands by consumers, $D = (D_1, \ldots, D_n)$ are functions of q. The arguments of the previous section suggested, and is rigorously demonstrated by Diamond and Mirrlees [1], that there are accounting prices $p = (p_1, \ldots, p_n)$ such that optimum production $x^* = (x_1^*, \ldots, x_n^*)$ maximizes

$$px = p_1x_1 + p_2x_2 + \ldots + p_nx_n.$$

If, when commodity taxes are optimally chosen, market prices are

$$q^* = (q_1^*, \ldots, q_n^*),$$

optimum production is

$$x^* = D(q^*). \tag{8}$$

It is shown by Diamond and Mirrlees [1] that the prices p can be chosen so that

$$\frac{\partial}{\partial q_i} V(q^*) = M \frac{\partial}{\partial q_i} \Sigma p_j D_j(q^*) \tag{9}$$

for a constnat M[8]. It is also shown that (9) can be interpreted as saying that

[7] *Cf.* [3 ; 7]. Methods for obtaining crude estimates of accounting prices are discussed in [5].

[8] Certain assumptions not mentioned here are required. They are rather technical, and not at all restrictive.

the marginal effects on welfare of increases in different tax rate should be proportional to the marginal increases in government revenue arising from the tax changes.

Suppose national income is changing. The change will be associated with change in market prices. We have:

$$\frac{d}{dt} V = \Sigma_i \frac{\partial}{\partial q_i} V(q^*)\dot{q}_i^* = M\Sigma_j p_j \left(\Sigma_i \frac{\partial}{\partial q_i} D_j(q^*)\dot{q}_i^* \right). \qquad (10)$$

From (8), we see that

$$\dot{x}_j^* = \Sigma_i \frac{\partial}{\partial q_i} D_j (q^*)\dot{q}_i^* .$$

Therefore,

$$\frac{d}{dt} V = M\Sigma_j p_j \dot{x}_j^*,$$

as we expect.

Relations (9) define the optimum relation between market prices and accounting prices. It will be noticed that these relations can hold even if income taxes, *etc.*, are not being chosen optimally. The conclusion (10) holds even if some of the market prices are fixed and constant for some reason, not deliberately chosen by government[9]. Thus, the conditions necessary for the conclusion that changes in national income at constant accounting prices reflect changes in welfare are that:

a) some commodity taxes are being chosen optimally;

b) all policies not chosen optimally, and all market prices not determined optimally, remain constant.

Of course in any comparison of national income over time, it has to be assumed that preferences and the circumstances of demand remain constant: this condition is less likely to be satisfied the more circumstances — *e.g.* income tax rates — have to be assumed constant, if not optimally determined.

IV

Our argument so far implies only that under certain rather utopian

[9]If a *tax rate* on some commodity remains constant, the producer price may change, and as a result the market price will change. In this case there is no guarantee that accounting-price national income is a good measure. But in some cases, *e.g.*, when there is a fixed tariff on a commodity imported at a constant price, the condition is satisfied.

circumstances namely, that the government uses whatever tools are at its disposal with infinite wisdom — is it possible, in principle, to use accounting prices to evaluate changes in welfare. Admittedly we can assume, alternatively, that policies not chosen optimally remain constant, but this is in practice an unreasonable assumption. Governments may have more or less good or admirable attentions; but they vary particular details of the tax structure only occasionally, and when changes are made, they are often dictated by considerations of immediate expediency. Policies are certainly not (yet) calculated to obey the rules implied by a particular social welfare function. It seems that, if a measure of the changes in social welfare is wanted, it will have to be constructed directly from information about the changes in consumption of different income groups.

It does seem to me that national income measured in terms of certain accounting prices may nevertheless be given an important interpretation. We may attempt to estimate accounting prices in such a way that a small change in income at these prices shows the change in *maximum attainable social welfare*, even when the government is not pursuing policies that allow social welfare to be maximized (given the instruments at its disposal). For this purpose, it is necessary to estimate what the accounting prices would be if available production capacity in the economy were being used in an optimal manner, so as to attain the optimum attainable state of the economy. In the earlier discussion, the methods by which it was thought accounting price might be estimated were perfectly appropriate to this end. Many of the prices are determined by trading possibilities; others depend on a more subtle analysis, perhaps by means of optimizing models, in which welfare considerations may appear directly. At first sight, it is hard to see even how accounting prices can be defined in a non-optimal situation (although I think that is not necessarily so hard); but in fact there would seem to be some advantage in attempting to estimate the 'optimal' accounting prices.

Once this is done, we may imagine that we are decomposing the *actual* change in social welfare into two parts; the change that results from a change in *attainable* welfare; and the change that is attributable to changes in the extent to which actual taxes, tariffs, quotas and controls reduce social welfare below the maximum. A new textile mill should bring about an increase in social welfare; it will — or might — do so by increasing the value of the economy's production in world-price terms; or it might do so by ameliorating the adverse effects of a non-optimal tax on imports of textiles. It does not seem reasonable to regard this second effect as an increase in the economy's production, since the increase in welfare could in fact have been obtained without the new textile mill having been set up. The fact that the desirable tariff change has not been made will usually be the result of inadvertance or sectional pressures. If the adverse effects have been lessened, that is good, but in describing the cause of

the economy's development, it is as well to separate this effect from the straightforward benefit of having extra output available, as a result of increasing the economy's productive capacity.

There can be no unambiguously right way of evaluating the national income, or the contribution of particular sectors to the national income. When an investment project is evaluated, it may be necessary to make alternative recommendations: it would be best, say, to remove an import quota on fertilizers (while temporarily restricting aggregate investment); but if that will not be done, it would be a good idea to set up a fertilizer factory. In such a case, the economist should also point out the cost of adopting the less satisfactory plan. Similarly, one cannot measure the contribution of a particular sector to the national income unless one knows what would have remained the same — particularly taxes, rations, quotas and the like — if activity in that sector had been different. It seems best not to assume that circumstances are fixed and unchangeable if the government could perfectly well change them. But it is hard to draw a clear line between what the government can do and what it cannot. The suggestion that national income statisticians at least assume that commodity taxes could have been different has the advantage, apart from the plausibility of the presumption, that the implied accounting prices can perhaps be estimated[10].

V

It is unfortunate that, in an imperfect economy, the prices appropriate to measuring the national income are not actual prices, and therefore cannot be observed directly. They are surely not, in any presently existing economy, market prices, for reasons we have already seen. They are not, in general, factor prices. Factor prices would indeed be the appropriate prices to use if control of the markets for goods and services were exercised entirely by means of uniform taxes. But this requires, in particular, that tariffs and export taxes different from 'optimum' taxes and tariffs are not levied. Revenue or protective taxation of foreign trade is ruled out. Undeniably, it is awkward to want an economic statistics that is not defined by standard procedures applied to observable data: the 'objectivity' of the statistics appears to be compromised. But there is no way out. Either one has a (relatively) meaningless figure, or one employs economic expertise and guesswork in the construction of the statistics. We should not have expected anything else.

The arguments of this paper suggest that the situation and development

[10]Dr. A. R. Khan and I are currently engaged in attempts to estimate accounting prices for the economy of Pakistan, and to compute national income and its sectoral and regional components at accounting prices.

of an economy might be rather informatively portrayed by means of the following scheme of estimates:

	Income (accounting prices)	Income (market prices)	Indirect taxes + profit taxes	Income group I	Income group II	Income group III
Sectors of the economy						
Total						

Net income: Tax + + = (market prices)

From abroad GNP : Consumption: + =

Net borrowing from abroad: „ + + = (accounting prices)

Public consumption =

Gross capital formation =

= Gross National Expenditure (accounting prices) =

The different income groups listed in the table might be wage-earners, profit-earners, and self-employed people, or (ideally) groups classified by size of income. It must be admitted that it is bound to be hard to estimate the details of these columns, interesting though they are.

Certain points must be made about the interpretation of such a form of national accounts. First, it should be emphasized that comparison between a particular component measured in market prices and the same component measured in accounting prices tells one nothing. Accounting prices could all be doubled, and would still be perfectly correct accounting prices. The theory only claims significance for *changes* in items, measured at constant prices, and small ones at that. It is perhaps not extremely misleading to compare, say, the contributions of two sectors, each measured by means of accounting prices, and to remark upon the relative importance of these sectors. But a comparison between items measured in different prices has no meaning. In particular, one cannot estimate, in an absolute sense, the subsidy that a sector 'really' receives, the difference between its actual income and its accounting price. A doubling of all accounting prices would change all apparent taxes and subsidies, but change none of the correct deductions from the figures. One can, of course, estimate the difference between the contribution of a sector to production on

the one hand, and the value, in accounting prices, of the consumption (and possibly, asset-acquisition) of those who receive the incomes of the sector: this might be interpreted as the true tax or subsidy on the sector, but is very hard to estimate.

Secondly, public consumption and capital formation should be measured in accounting prices too. This is really a matter of convention: their value, compared to the value of consumption, might well be substantially greater than (or, at any rate, different from) the value suggested by the figures in the table. For that value derives from the final value of the results of these expenditures for households, and on that no direct evidence has been brought to bear. Only if the government's expenditure policies and the level of saving in the economy are optimal can changes in the value of these items be taken to be measures of social welfare change comparable to changes in consumption. But it is convenient to use the same prices; and it is an advantage that, in principle, a small reduction in consumption, measured at accounting prices, would make possible an equal increase in, say, investment; and *vice versa.*

Finally, one might supplement the picture of the economy's development given by the table for a number of years, by computing rough estimates of changes in social welfare from year to year. To do this, one must weight changes in the consumption of different income groups by estimate of the marginal welfares of average members of the groups. Weights inversely proportional to average income per head (after tax), or to the square of average income per head, might be appropriate. It must be recognized, however, that the empirical basis for the components of such an average is likely to be very weak at present, quite apart from the supposedly embarrassing precision of the value judgment employed.

VI

The method of evaluating national income explored in the paper, if satisfactory, is so partly because accounting prices in many economies may not be very sensitive to the particular assessment of social welfare. A direct evaluation of changes in social welfare, in terms of the consumption of different income groups, is hard to do, and may depend quite sensitively on the sepcification of the welfare function. In any case, the practical significance of national income changes evaluated by means of accounting prices is considerable; since, in a natural sense, these changes measure the 'achievements' of the economy, considered as a mechanism for production. If growth rates are to be used for grading governments, I should like to see them measured at accounting prices.

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