

Interrelationships of Some Fertility Measures in Pakistan

by

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Unlike mortality or migration, the fertility behaviour of a population largely determines its age distribution¹. A high fertility population maintains a broad-based age pyramid by adding continuously a large number of persons at the first year of life. In such a population the dependency ratio of children (say persons under 15 years) remains high relative to the size of its working age population (say those aged 15-64 years). A decline in fertility reduces this ratio and restructures the age distribution to make it more favourable to economic growth² [4]. For this reason the study of human fertility occupies a singularly important position in the demographic literature today.

In this paper the fertility of the population of Pakistan is examined. The purpose of the paper is to evaluate several fertility measures obtained from data collected through the Population Growth Estimation project in Pakistan³. The interrelationships of these measures in terms of their practical utility to planners for action programmes will be examined. Further, the possible impact on fertility resulting from a rise in the age at marriage in Pakistan will be discussed. An effort will also be made to identify the demographic differentials, relevant to planners, in the two provinces of the country. To simplify the presenta-

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¹ A proportional improvement of mortality at all ages, for example, would not change the age distribution at all, while the effect of an improvement in infant mortality from a high initial level is similar to the effect produced by a rise in fertility. The amount of net flow of international migration is highly limited these days, while internal migration does not affect the age distribution of a national population at all.

² Of course, very low fertility levels along with low mortality levels continued for a long period of time result in an aging population. This may cause other economic problems, *i.e.* an increasing dependency of the aged persons.

³ Unlike vital statistics collected through other sources in Pakistan, data from this source are found to be, in analytical studies, internally consistent and on *a priori* grounds, quite reliable. For the technical details of the project and discussion of some of its empirical findings, see [1; 2; 6; 7; 8; 9; 10; 11].

tion, technical definitions and calculation procedures for obtaining the demographic measures are shown in the Appendix.

Data used in this article were obtained by the Population Growth Estimation project during 1962 and 1963⁴. The rates used in this report are limited to the Chandrasekar-Deming estimates which are the result of a combination of two methods of data collection: continuous longitudinal registration by full time resident registrars and quarterly cross-sectional surveys⁵.

Mortality rates are based on aggregated 1962 and 1963 data. Since births by age of mother were available for 1963 only, fertility and birth rates refer only to 1963. Birth rates were adjusted, however, to correct for what appears to have been an underenumeration/registration of female births in West Pakistan. In West Pakistan, 109.1 males were reported to have been born per 100.0 females. To adjust, the 105.7 sex ratio at birth recorded for East Pakistan was taken as the standard, and the number of female births in West Pakistan inflated and distributed proportionately by age of mother⁶.

The Fertility Measures

The fertility rates for East, West and all Pakistan by sex for the year 1963 are shown in Table I. The age-specific fertility rates for both sexes ($ASFR_{bs}$) and for females only ($ASFR_f$) are also plotted in Figure 1. The fertility level as measured by the total fertility rate (TFR) is slightly higher in West Pakistan (7.99) than in East Pakistan (7.89), though the reverse is true for the gross reproduction rate (GRR), 3.87 for East and 3.84 for West Pakistan.

These variations in the total fertility rates and gross as well as net reproduction rates stem from differences in the age-sex fertility schedules in the two provinces of Pakistan. At older ages of women, one birth carries relatively more weight in the calculation of fertility rates than a birth at earlier ages because the denominators of the age-specific fertility rates (number of women at various ages) decrease with advanced ages. Putting the case in a different way, one may cite a theoretical example. Given the same number of total births and the same age distribution of women, a population with an ascending order of births with advanced ages of women would produce a higher fertility rate than a population with a descending order of births⁷.

⁴ The data collected are based upon a sample survey covering selected areas in East and West Pakistan and are, of course, subject to sampling errors as well as non-sampling errors.

⁵ For a description of the PGE methodology, see [1].

⁶ This adjustment procedure for sex imbalances at birth raised the crude birth rate in West and all Pakistan by 0.8 points and 0.4 points respectively. Its effect on GRR and TFR is to raise each of these rates by 0.1160 in West Pakistan and 0.0497 in all Pakistan.

⁷ In this paper a clear distinction between the fertility rate and birth rate has been rigidly maintained. For the meanings of the terms, see the Appendix.

Empirically, the situation may be clarified by comparing the births in various age groups of women in East and West Pakistan in Table II. It can be seen from this table that the proportions of births at older ages are higher and at younger ages lower in West Pakistan as compared to East Pakistan. This is exactly why the total fertility rate in West Pakistan is slightly higher than (or nearly the same as) that in East Pakistan, despite the fact that birth rate level is about 5 points higher in East Pakistan than that in West Pakistan⁸. Thus, it may be misleading to suggest that in comparison to a birth rate, a total fertility rate (TFR) or gross reproduction rate (GRR) is a superior estimate of fertility, even though, unlike the birth rate, the TFR or GRR takes into account age differentials in the fertility of women.

In order to understand the relationships between the birth rate and the TFR and GRR the latter two should be studied in conjunction with the mean age of childbearing (T) of a population. This is especially important when the rate of growth of a population is high [5, p.30].

Given the same fertility and mortality rates, a population with a higher mean age of childbearing would produce a lower birth rate. The mean ages of childbearing for East and West Pakistan are calculated to be 26.3 years and 28.8 years respectively (see Appendix for calculation procedure). These data, therefore, provide further evidence of the actual gap between East and West Pakistan with regard to birth levels. Two other methods may be used to evaluate these differences: first, given the GRR values of East and West Pakistan, it can be calculated, by interpolation/extrapolation from the Princeton *Stable Population, Females, "GRR" Set* corresponding to appropriate model life tables⁹ that the stable birth rate of East Pakistan is around 56 and that of West Pakistan around 51, confirming the previous findings.

⁸ The real or intrinsic birth rates as obtained from the PGE fertility (1963) and mortality (1962-63) schedules are:

Country or province	Birth rate per 1000 population		
	Both sexes*	Males	Females
All Pakistan	53.42	52.92	53.95
East Pakistan	55.68	55.36	56.03
West Pakistan	50.76	50.09	51.48

* Calculated from the formula

$$\frac{(1+u)(b_f b_m)}{ub_f + b_m}$$

Where b_f = female birth rate

b_m = male birth rate

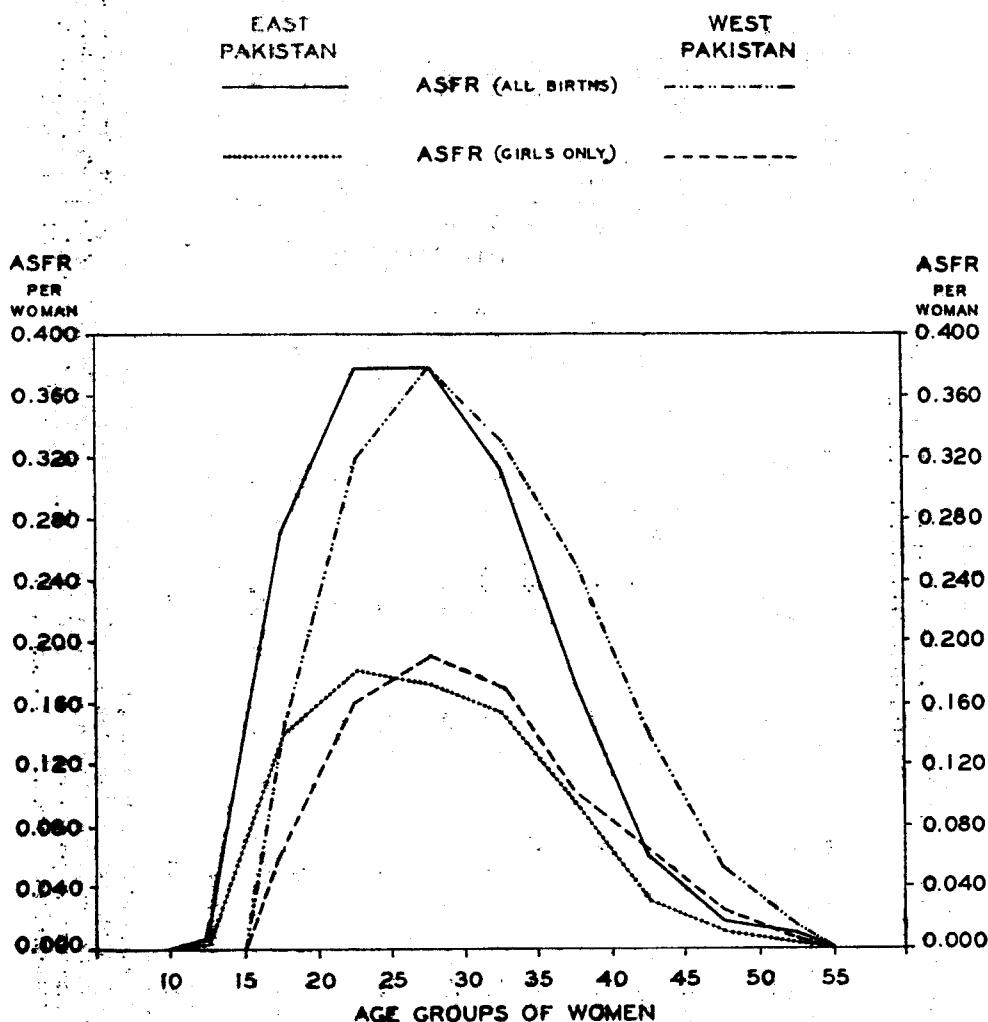
u = sex ratio at birth which is set at 1.057.

⁹ The assumed appropriate model life tables are for East Pakistan females' Princeton Model "East" Level 11 and for West Pakistan females, Model "South" Level 12 [5].

TABLE I
TOTAL FERTILITY, GROSS REPRODUCTION AND NET REPRODUCTION RATES
PAKISTAN AND PROVINCES, 1963

Age intervals of women	ASFR _{bs}			ASFR _c			(ASFR _c)(sL_x/l_0)		
	All Pakistan	East Pakistan	West Pakistan	All Pakistan	East Pakistan	West Pakistan	All Pakistan	East Pakistan	West Pakistan
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
10-14	.00318	.00548	—	.00191	.00329	—	.00716	.01254	—
15-19	.21059	.26753	.13727	.10333	.13625	.06222	.38064	.51058	.22404
20-24	.34879	.37377	.31563	.17000	.17752	.15936	.61012	.64683	.56056
25-29	.37456	.37382	.37555	.17823	.16996	.18873	.61974	.59717	.64728
30-34	.31577	.30828	.32551	.15838	.15152	.16730	.53115	.51117	.55665
35-39	.20230	.16797	.24695	.09562	.09262	.09959	.30762	.29876	.31930
40-44	.08815	.05641	.13477	.04270	.02937	.06229	.13118	.09009	.19186
45-49	.03024	.01634	.05031	.01609	.01089	.02358	.04700	.03157	.06968
50-54	.01012	.00879	.01220	.00298	.00195	.00457	.00824	.00531	.01293
	1.58370	1.57839	1.59819	0.76924	0.77337	0.76764	2.64285	2.70402	2.58230
Total Fertility Rate			7.92	7.89	7.99	Gross Reproduction Rate			
						3.85	3.87	3.84	
							Net Reproduction Rate		
							2.64	2.70	2.58

Note: Birth statistics are based on Chandrasekar-Deming estimates obtained from the Population Growth Estimation project. Births under 15 years of age of mothers are assumed to fall in the age group 10-14 and those above 50 in the age group 50-54. Births and population of ages unknown are allocated *pro-rata* in all age groups. PGE (adjusted) mid-year population is used for calculating fertility rates. Births are adjusted for underreporting/enumeration of female births in West Pakistan to correspond to the sex ratio at birth of 1.057. The factor sL_x/l_0 is obtained from the respective life tables in [3].



SOURCE: TABLE 3

Figure 1. Age-Specific Fertility Rates in East and West Pakistan, 1963.

TABLE II

BIRTHS IN VARIOUS AGE GROUPS OF WOMEN IN EAST AND WEST PAKISTAN COMPARED, 1963

Age groups of women	Number of births (in 000's)		Per cent of all births		
	East Pakistan	West Pakistan	East Pakistan	West Pakistan	Difference (5) — (4)
(1)	(2)	(3)	(4)	(5)	(6)
All births	2,985	2,150	100.0	100.0	—
10-14	15	—	0.5	—	—0.5
15-19	538	214	18.0	10.0	—8.0
20-24	798	511	26.7	23.8	—2.9
25-29	794	593	26.6	27.6	+1.0
30-34	529	430	17.7	20.0	+2.3
35-39	214	243	7.2	11.3	+4.1
40-44	73	119	2.4	5.5	+3.1
45-49	9	8	0.3	0.4	+0.1
50-54	15	32	0.5	1.5	+1.0

Note: Births (both sexes) are based on Chandrasekar-Deming estimates obtained from the Population Growth Estimation project. Births to mothers of unknown ages are allocated *pro-rata* in the age groups of women. Figures are rounded independently.

Secondly, the intrinsic rates of growth of population (r) are calculated to be 0.03787 and 0.03295 for East Pakistan and West Pakistan respectively (see Appendix for calculation procedure). Since the East and West Pakistan death rates under the PGE system are at about the same level¹⁰, the 4.92 point difference in the values of r per 1000 population in the two provinces would further confirm our observation regarding the difference in the birth rate levels in the two wings of the country.

The Meanings of Fertility Rates

An attempt will be made here to provide a theoretical meaning of the total fertility rate (TFR), gross reproduction rate (GRR) and net reproduction rate (NRR) values in Pakistan. This will be done along traditional lines and with the rates for all Pakistan only¹¹.

¹⁰ Crude death rates based on Chandrasekar-Deming estimates under the PGE system are 19.6 for East and 19.1 for West Pakistan in 1962-63.

¹¹ For the meanings of a similar rate for a province, the interested reader may substitute the provincial values for corresponding values for all Pakistan in the analytical descriptions of this section.

A 1963 TFR of 7.92 (Table I) simply indicates that a woman in Pakistan, during the whole childbearing period, on the average, gave birth to this many children. In the same sense, a GRR of 3.85 would indicate that a similar woman, on the average, produced 3.85 female babies. In generation terms, however, assuming the fertility conditions of 1963 prevail and no woman from birth to and during the childbearing period dies, the above figures would suggest that a woman at the completion of her childbearing period would give birth, on the average, to 7.92 babies of which 3.85 would be girls. Since the assumption of immortality of women from birth to and during the childbearing period is unrealistic, the generation concept of the values of TFR and GRR is modified by introducing the influence of mortality. Traditionally such an exercise is done with respect to female births only. Mortality conditions of a woman from birth to the end of the childbearing period, when applied to the generation concept of GRR, are reflected in the net reproduction rate (NRR).

The NRR of 2.64 suggests that a Pakistani woman now passing through the childbearing period, after an allowance for mortality from birth to the end of the childbearing period is made, would replace herself, on the average, by 2.64 similar women in the next generation¹². In interpreting this value, the underlying assumptions in its calculation should be clearly noted. *Explicitly* it is assumed that fertility as of 1963 and mortality as of 1962-63 will remain unchanged in the course of a generation. *Implicit* assumptions are that the nuptiality rate, age at marriage, birth parity patterns, *etc.*, will remain unchanged in the course of the next generation. In a rapidly changing society like Pakistani, it is highly unlikely that the explicit and implicit assumptions underlying the calculation of NRR will remain valid in the future. Mortality in Pakistan is declining [2] and fertility is also expected to decline in the future given the continuance and further intensification of family planning efforts in Pakistan.

Theoretically a population with a net reproduction rate of 1.00 would replace itself, neither growing nor declining, in the next generation. The NRR of 2.64, therefore, indicates that births are in excess of the number needed for simple replacement by more than 160 per cent. Another way of viewing the impact of current fertility under 1962-63 mortality conditions is to calculate the total number of children a woman in Pakistan need produce to maintain the present size of the population. These calculations show that an average of 3.0 births per woman¹³ would maintain the present population, implying that the fertility

¹² The average lengths of generation for East and West Pakistan are calculated to be 26.3 and 28.8 years respectively (see Appendix).

¹³ The figure is obtained by relating the total fertility rate with net reproduction rate as under:

$$\begin{array}{ccc} \text{NRR} & \text{TER} & \\ 2.64 : 1.00 = 7.92 : x & \therefore x = 7.92/2.64 = 3.00. \end{array}$$

needs to be reduced to about 38 per cent of the present level. An average of 3.0 births would, of course, in the future be too high since mortality conditions may be expected to improve.

The Possible Impact of Raising the Age at Marriage on the Birth Rate

It has been pointed out that given the same fertility rates, an increase in the mean age of fertility schedule (T) would decrease the birth rate, other things remaining the same. T can be increased by raising the age at marriage. It can be calculated that if the fertility schedules of East and West Pakistan as of 1963 are shifted five years forward, total fertility rates remaining the same, the birth rates would be reduced by about 11 points per 1000 persons in East Pakistan and by about 9 points in West Pakistan¹⁴. Shifting the fertility schedule forward but keeping the shape of this function unchanged would be somewhat unrealistic. Since the fertility schedules in both provinces increase up to the age group 25-29, and then fall off with advanced ages, shifting the schedule five years forward really means lowering age-specific fertility rates up to the age group 25-29 and raising such rates for women in the age group 30-34 and above. Given an already high fertility rate, it is unlikely that with an increase of age at marriage, fertility rates of women 30 years and above would increase substantially. If, as an extension of our above calculation, it is further assumed that fertility rates for women 30 years and over would not increase, the overall reduction in the birth rates would be roughly 18 points per 1000 persons in East Pakistan and around 16 points in West Pakistan: these reductions amount to 31 per cent in East Pakistan and 30 per cent in West Pakistan. These are very impressive reductions, indeed, and result from our optimistic assumptions regarding the improvement in the minimum age at marriage and the shape of the fertility schedule after the age at marriage would have increased. It is, however, quite possible that with an increase in the age at marriage, fertility at ages immediately after the marriage would rise to compensate for births lost by delayed marriage. This phenomenon would depress the possible reduction in birth rate somewhat from the levels shown above.

The point to be made here is that with an increase in the minimum age at marriage a very substantial decline in birth rate may be expected. However, since East Pakistan women start producing children earlier than their West Pakistani counterparts, an equal increase in the average age at marriage in East and West Pakistan would reduce the birth rate more in East than in West Pakistan.

¹⁴ In order to eliminate the disturbing effect of the irregular age distribution of Pakistan population, the calculations in this section have been made on stable populations obtained from PGE fertility and mortality schedules. For further details on this, see [8]. The fertility schedule referred to in this section is the age-specific fertility rates for both sexes.

Summary and Conclusions

Until to date in Pakistan little empirical information has been available on fertility, largely as the result of the absence of vital registration information for the country as a whole or for significant sections of the country. In the absence of such information a variety of estimates of fertility have been made, generally using stable population theory. The results have not been consistent.

Beginning in 1962 the Population Growth Estimation project, a joint venture of the Pakistan Institute of Development Economics and the Central Statistical Office, began collecting data on a sample basis in order to generate mortality and fertility estimates for the country. Data collected in 1963 provided age-specific fertility rates for the country as a whole and for East and West Pakistan separately. Using this information as well as information on mortality for 1962 and 1963 a variety of birth and fertility measures have been generated. Any and all measures indicated that fertility in Pakistan and the consequent growth rate of the population are extremely high.

The total fertility rate in Pakistan is approximately 7.9 and the gross reproduction rate is 3.8. Given these rates it is estimated that the population of Pakistan, under 1962-63 mortality conditions, is replacing itself at a rate more than 160 per cent higher than what is required to maintain the population at its present level. To maintain the population at a stationary level (rate of growth = 0) the necessary number of children an average Pakistani female need bear is only 3.0. In order to reach this stationary level, fertility needs to be reduced to about 38 per cent of the present level.

These estimates reflect certain assumptions which probably will not continue to hold. First, it is assumed that mortality conditions would remain unchanged from the 1962-63 level. Mortality has been declining rapidly in Pakistan and it is unlikely that the trend will not be continued, following the pattern of other developing nations. Second, the calculations assume a constant age at marriage. Any rise in the age at marriage could reduce the number of births. An increase of five years in the average age at marriage could reduce the birth rates by 31 per cent in East Pakistan and 30 per cent in West Pakistan. However, such a radical increase in the age at marriage, in the short run, is unlikely.

The data made available by the Population Growth Estimation project for the years covered in this study suggest that fertility conditions and growth rates differ between East and West Pakistan. Although the fertility rates are roughly the same in the two provinces, the true or intrinsic birth rate in East Pakistan is about 5 points higher than in West Pakistan. In part, the difference is due to the lower mean age of childbearing in East Pakistan, 26.3 years *versus* 28.8 years in

West Pakistan. The intrinsic rates of growth are 3.8 per cent in East Pakistan and 3.3 per cent in West Pakistan.

Since the data are based on a sample survey, sampling errors as well as non-sampling errors may explain part of the observed provincial differences. For example, the case of the East-West Pakistan difference in the sex ratio at birth is assumed to be due to greater underenumeration of females at birth in West Pakistan. Data for 1964 and 1965 which will be available for publication soon should provide additional insights in the fertility performance and mortality rates in Pakistan and its provinces.

The implications of the study are clear. The corollary of a high fertility is a *young population* with a high burden of children relative to the working-age population. A high dependency burden exerts a continued pressure for consumption at the cost of savings. High fertility also reduces the number of women available for both household work and gainful employment. Declining mortality would intensify the process making the attainment of such Perspective Plan targets as universal literacy by 1985 *etc.*, more and more costly. While the growth rate may be reduced by an increase in the age of marriage, it is unlikely that the age of marriage will rise significantly in the next few years. Of more importance will be the continued emphasis on and expansion of family planning programmes and facilities.

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Appendix A

A *crude birth rate (CBR)* is simply a ratio of total births in a period to the average size of the population during the period, usually the size at the mid-period. The rate is usually expressed as births per 1000 such persons. A true or real or *intrinsic birth rate* (or death rate or rate of growth) is the rate that a population would ultimately attain, if a particular fertility and mortality patterns remain unchanged for a considerable period of time. It should be noted that an intrinsic rate, as is usually calculated from data for a given year or so, may not be exactly equal to the one that the population would ultimately attain, even with the continuance of the same fertility and mortality schedules.

A *fertility rate*, as opposed to a birth rate, incorporates the variations in the childbearing performance of women in various age groups. The age-specific fertility rate is the proportion of women at a particular age (or age group) producing a live birth during a time period, usually a year. The age-specific fertility rates summed over all ages, from the beginning to the end of childbearing period of an average woman is the *Total Fertility Rate* or *TFR*. The total of such rates referring to female births only is the *Gross Reproduction Rate* or *GRR*. These two rates—*TFR* and *GRR*—when used in a generation sense would indicate the average number of births (*TFR*) and female births only (*GRR*) born to an average woman. These measures make no allowance for mortality; it is assumed that all women live from birth to the end of their childbearing period. When the influence of mortality is introduced in the above concept of *GRR*, the result is the *Net Reproduction Rate* or *NRR*.

In symbolic and integral terms, if $f(a)$ is the age-specific fertility rate (referring to all births to female population), $m(a)$ is the fertility schedule (referring to female births only to female population)¹, $c(a)$ is the proportion of women at age a to the total population, α and β are the lower and upper limits of childbearing period, w is the highest age attained, r is the intrinsic growth rate and $p(a)$ is the proportion surviving from birth to age a , then

¹ The calculations in this Appendix for obtaining the values of *GRR*, stable rates, and the mean age of childbearing have been made on female population only (relating female births born to them) though a similar exercise can be made relating male births to male population also.

$$CBR = \int_{\alpha}^{\beta} c(a) f(a) da \quad (1)$$

$$\text{Intrinsic birth rate} = \frac{1}{\int_0^w e^{-ra} p(a) d(a)} \quad (2)$$

$$TFR = \int_{\alpha}^{\beta} f(a) da \quad (3)$$

$$GRR = \int_{\alpha}^{\beta} m(a) da, \quad (4)$$

and

$$NRR = \int_{\alpha}^{\beta} m(a) p(a) da \quad (5)$$

The formulae are based on the notion of infinitesimal intervals of age, and need to be revised for computations with the usually available 5-year age group data. These revisions have been used in the calculations of this paper and are:

$$CBR = 5 \sum_{x=\alpha}^{\beta-5} c(x+2.5) f(x+2.5) \quad (1a)$$

$$\text{Intrinsic birth rate} = \frac{l_0}{\sum_{x=0}^{w-5} l_x e^{-rx}} \quad (2a)$$

$$TFR = 5 \sum_{x=\alpha}^{\beta-5} f(x+2.5) \quad (3a)$$

$$GRR = 5 \sum_{x=\alpha}^{\beta-5} m(x+2.5) \quad (4a)$$

$$NRR = 5 \sum_{x=\alpha}^{\beta-5} m(x+2.5) \frac{5L_x}{l_0} \quad (5a)$$

where l_0 is the radix of a life table and $5L_x$ is person-years lived between ages x and $x + 5$.

Calculation of r , the Intrinsic Rate of Growth

The value of r , as implicit in the fertility and mortality schedules, can be calculated in the direct method (as originally done by Lotka, [5a]), or by a recursive method (as shown by Coale [3a]). The latter method was adopted in the calculation of r in this Appendix because it provides accurate results and is simpler to use. (A similar exercise can also be found with Chilean data in [1a, pp. 216-222]). In essence the method, by applying the stable population concepts, first obtains an approximate value of r (shown as r') from an approximate value of T (shown as T'), the mean length of childbearing. Then a correction factor is applied to rectify the deviation of the calculated value from expected value in the following steps:

Step 1. In a stable population, NRR is related to r , the stable rate of growth, through T , the mean length of childbearing, in the following way:

$$\text{NRR} = e^{rT} \quad (6)$$

An approximate value of T can be obtained by multiplying the number of expected female births in the next generation in each age group of women (Col. (3), Table A-1) by the mid-point of each age group (Col. (2), Table A-1), summing the values and dividing the total by the expected female births per woman in the next generation. This value for East Pakistan females is 27.17859. With this approximate value of T , an approximate value of r can be obtained from equation (6). This approximate value of r is 0.03660 for East Pakistan females.

TABLE A-1
CALCULATIONS FOR OBTAINING THE INTRINSIC RATE OF GROWTH(r)
FOR EAST PAKISTAN FEMALES

Age group of women x to $x + n$		Calculations for T'			$r' = 0.03660$	
		Mid-point of age group (a)	Expected female births in next generation $m(a)$ $p(a)$	Columns (2) $\times$ (3)	$e^{-r'a}$	$e^{-r'a} m(a) p(a)$
(1)		(2)	(3)*	(4)	(5)	(6)*
10-14	...	12.5	0.01254	0.15675	0.63286	0.00794
15-19	...	17.5	0.51058	8.93515	0.52703	0.26909
20-24	...	22.5	0.64683	14.55368	0.43889	0.28389
25-29	...	27.5	0.59717	16.42218	0.36550	0.21827
30-34	...	32.5	0.51117	16.61302	0.30437	0.15558
35-39	...	37.5	0.29876	11.20350	0.25347	0.07573
40-44	...	42.5	0.09009	3.82882	0.21108	0.01902
45-49	...	47.5	0.03157	1.49958	0.17578	0.00555
50-54	...	52.5	0.00531	0.27878	0.14639	0.00078
			2.70402	73.49146		1.03585

* $p(a)$, in life table terminology, is equal to ${}_5L_x/l_0$ and is taken from the Life Table for East Pakistan Females, 1962-63 in [3]; $m(a)$ is the fertility schedule (female births to female population, ASFR_f) and is taken from Table I.

The calculations for r and T are as under:

$$T' = \frac{73.49146}{2.70402} = 27.17859$$

$$NRR = e^{r' T'} = e^{r' (27.17859)} = 2.70402$$

$$r' = \frac{1}{27.17859} \log_e 2.70402 = 0.03660$$

$$\begin{aligned} 1.00000 + E &= 1.03585 \\ E &= 0.03585 \end{aligned}$$

$$r = r' + \frac{E}{T' + \frac{E}{r'}} = .03660 + .00127 = 0.03787$$

$$T = \frac{1}{r} \log_e NRR = \frac{0.99474}{0.03787} = 26.3 \text{ years}$$

Step 2. The basic equation of a stable population as developed by Lotka is as under:

$$\int_0^w e^{-ra} p(a) m(a) da = 1 \quad (7)$$

In this formula, o and w are the lowest and the highest limits of a life span; e^{-ra} is the growth factor; $p(a)$ is the proportion surviving from birth to age a ; and $m(a)$ is the fertility schedule relating female births to female population. In working with usual five-year age group data, equation (7) can be rewritten as:

$$\sum_{x=\alpha}^{B-5} e^{-r(x+2.5)} m(x+2.5) \frac{L_x}{l_0} = 1 \quad (8)$$

Applying the approximate value of r as obtained in Step 1 in the equation (8) above, we get a value of 1.03585 (Table A-1).

Step 3. The last value as obtained in Step 2 deviates from the expected value of 1.00000 by 0.03585 primarily as the result of using an approximate value of r in equation (8). Adjusting for this deviation of the observed value from the expected value (*see* the foot of Table A-1 for adjusting procedure), we derive the final value of r which is 0.03787 for East Pakistan females.

Applying this value of r in equation (6), we deduce the final value of T , the mean length of childbearing, which is 26.3 years for East Pakistan females. A similar exercise with data for West Pakistan would indicate the value of r and T to be 0.03295 and 28.8 years respectively.