A Multi-Sector Programming Model for Regional Planning in Pakistan

by

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1. INTRODUCTION

This article presents the outline of a multi-sector, optimizing model for inter-regional planning and uses it to analyse Pakistan's Third Five-Year Plan (1965-70). This is an interregional (as distinguished from a national) model in so far as it recognizes the existence of economic regions within the nation and explicitly takes into account the interregional trade flows. Another aspect of a multi-region economy that the model reflects is that in planning for optimization the model enables the planners to maximize some objective which is a function not only of the value of national income or consumption but also of their distribution between the regions.

This is a multi-sector or detailed planning model. The economy of each region is divided into a number of producing sectors. The model takes explicitly into account the intersectoral flows. In doing so, the model reflects the implications of the differences in regional technologies and behaviour. A given consumption target would mean different vectors of consumer goods in different regions because consumption patterns are different between regions. A given final demand vector would mean different sectoral output levels and different sectoral allocation of investment in the two regions because the technological relations are different. The model is able to spell out the implications of all these factors by relating the regional and sectoral allocation of investment and foreign exchange to location of demand.

The present model does not merely provide for the feasibility of the plan but also satisfies certain efficiency criteria. In other words, this is an optimizing model which attempts to find the 'best' possible pattern of the allocation of investment and foreign exchange resources between regions and among sectors, the 'best' being understood in the sense of maximizing a given preference function subject to the relevant constraints.

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2. NOTATION

In the present formulation of the model there are two regions which we call East (representing East Pakistan, denoted by E and in the superfixes by e) and West (representing West Pakistan, denoted by W and in the superfixes by w). The productive activity of each region is divided into the following seven sectors (the letters in the parentheses denote the corresponding sectors):

(a) Agriculture, animal husbandry and forestry, briefly referred to as agriculture.

(f) Food processing industries

(m) Engineering

(b) Construction including building materials

(p) Fuel and power

(r) Other manufactures and minerals

(s) Trade, transport and other services

The choice of the number of sectors has been dictated by the consideration that it should be large enough to show a reasonable degree of disaggregation and interrelation among important types of activities and that it should be few enough for us to be able to describe its working verbally.

The following notation is used for region E. Similar notation is used for W with appropriate changes in the superfixes.

X^e_i = Production of i (i = a, f, m, b, p, r, s)

X^e_{ij} = Amount of i used as current-input for the production of j (j = a, f, m, b, p, r, s)

X^{ew}_i = Export of i to region W

X^{we}_i = Import of i from region W

$X^{ee}_i = X^e_i - X^{ew}_i$ = The part of region E's production of i, 'used' by the region itself.

$Z^e_i = X^{ee}_i + X^{we}_i = X^e_i - X^{ew}_i + X^{we}_i$ = Total "domestic supply" of i (*i.e.* that part of regional supply which originates in either of the regions).

Y^e_j = Value-added in sector j

$Y = \sum_j Y^e_j$ = Gross regional product

The above symbols denote *changes* in the corresponding variables over five-year period.

C_i^e = Private consumption of i

G_i^e = Government consumption of i

I_i^e = Fixed net investment of i

S_i^e = Increase in working capital requirement of i

D_i^e = Replacement requirement of i

E_i^e = Export of i abroad

N_i^e = Import of i from abroad

When put before C_i^e through N_i^e, the symbol d denotes changes over five-year period. Symbols without suffix i denote corresponding vectors, for example, dC^e is the vector of increase in consumption of goods of all sectors in region E over the five-year period. Symbols without superfixes e or w denote corresponding totals of two regions, for example, $dC = dC^e + dC^w$ and $Y = Y^e + Y^w$.

3. THE CONSTRAINTS OF THE MODEL

3.1 The Balance Equations

Our first set of constraints refers to the balance between availability and use of the products of each sector in each region. We write below the equations for region E. For region W the corresponding equations can be obtained with appropriate changes in the superfixes.

The first equation states the simple identity that regional domestic supply of i sector's output is parcelled out to intermediate use as current input of all sectors, net fixed investment, increase in working capital, replacement, private and public consumption and net exports abroad (all of the same region measured in terms of increases).

$$\text{(i)} \quad Z_i^e = \sum_j X_{ij}^e + dI_i^e + dS_i^e + dC_i^e + dG_i^e + dD_i^e + dE_i^e - dN_i^e$$

We express the current input demand for i in sector j as linear function of the level of output in sector j. This is the standard Leontief assumption of fixed input coefficients. Note, however, that we need not necessarily assume simple proportionality since our variables denote increases.

$$\text{(ii)} \quad X_{ij}^e = a_{ij}^e X_j^e$$

Import of i from the other region to this region is assumed to be a linear function of this region's domestic supply of i sector's output. Similarly the part of the domestic supply of i obtained by region E from itself is assumed to be a linear function of region E's domestic supply of i sector's output.

$$\text{(iii)} \quad \left.\begin{aligned} X_i^{we} &= r_i^{we} Z_i^e \\ X_i^{ee} &= r_i^{ee} Z_i^e \end{aligned}\right\} \quad (r_i^{we} + r_i^{ee}) = 1$$

This, or some variant of this, has been a standard assumption of a number of interregional input-output models[1]. This, however, amounts to the assumption that the entire volume of interregional trade is 'non-competitive'. If one considers potential import-substitutability this is certainly not an entirely satisfactory representation of the character of trade between the two regions of Pakistan. But given the present structures of the regional economies, they are mostly non-competitive. East's exports to West consist mainly of such goods as jute manufactures, tea, newsprint, paper and matches which are not produced in West now and are unlikely to be produced in large quantity in future. West's exports to East consist largely of raw cotton, cotton textiles and grains such as wheat. These are not being produced in East at the moment (except for a relatively small amount of cotton textiles) and are unlikely to be produced in large quantity (again except possibly for cotton textiles) in future. This relative non-competitiveness seems to be the main explanation of the fact that their coefficients have remained stable in Pakistan over relatively long periods of time[2]. Another way to justify the assumption is to regard the r's as the "planned" coefficients based on partial studies and *a priori* reasoning about interregional comparative advantage[3].

Next we have the identity that regional output of i is either "absorbed" within the region or exported to be "absorbed" in the other region.

(iv) $X_i^e = X_i^{ee} + X_i^{ew}$

Substituting equations (iii), (ii) and (i) in turn into (iv) we get:

$$\text{(v)} \quad X_i^e = \sum_j r_i^{ee} a_{ij}^e X_j^e + \sum_j r_i^{ew} a_{ij}^w X_j^w + r_i^{ee}(dI_i^e + dS_i^e + dC_i^e + dG_i^e + dE_i^e - dN_i^e + dD_i^e) + r_i^{ew}(dI_i^w + dS_i^w + dC_i^w + dG_i^w + dE_i^w - dN_i^w + dD_i^w)$$

In order to express investment requirement endogenously we assume that any new output would require additional capacity which has to be created through additional net investment. This can be justified by assuming that either there is no excess capacity or that the present rate of capital-utilization is normal in some sense and is likely to obtain in future. It is easily recognised that this assumption is of doubtful validity but is necessary if the problem imposed by the complete absence of information about the rates of utilisation is to be avoided.

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1 *See* for example [3, ch. 4]. Similar but not identical assumptions are made by [8] and [5].

2 These coefficients for 1959/60 and for 1964/65 are pretty much the same. These coefficients are likely to be more stable when we have only two regions than when we have more because in the two-region case the source of supply and the origin of demand are less in doubt.

3 Chenery treats similar coefficients as policy variables in [2]. It should be intuitively obvious that a change in these coefficients would result in a change in the required pattern of interregional allocation of investment and output.

The amount of investment of new fixed capital, by industry-of-origin, is a linear function of the expansion of outputs of the producing sectors : $I_i^e = \sum_j b_{ij}^e X_j^e$, where b_{ij}^e is the fixed capital coefficient. But in our balance equations (v) we have changes in investment (dI and dS) and not total investment over the five-year period to which I refers. In order to obtain dI from I we make the assumption, following Jan Sandee[4], that investment rises by an amount Q every year: $I_{it}^e = I_{io}^e + Qt$ (where I_{it}^e and I_{io}^e are respectively I_i^e in year t and base-year). Thus $I_i^e = \sum_{t=1}^{5} I_{it}^e = 5\, I_{io}^e + 15Q$ or $5Q = dI_i^e = 1/3\, I_i^e - 5/3\, I_{io}^e$. We therefore have an equation for dI_i^e and a similar equation for dS_i^e.

(vi) $dI_i^e = 1/3 \sum_j b_{ij}^e X_j^e - 5/3\, I_{io}^e$

(vii) $dS_i^e = 1/3 \sum_j s_{ij}^e X_j^e - 5/3\, S_{io}^e$

Summing these two equations we obtain

(viii) $dI_i^e + dS_i^e = 1/3 \sum_j k_{ij}^e X_j^e - 5/3\, I'^e_{io}$

where $k_{ij}^e = b_{ij}^e + s_{ij}^e$ and $I'^e_{io} = I_{io}^e + S_{io}^e$

In obtaining equations (vi) and (vii), we have related investment in a fiscal year to increase in output in the same fiscal year. In order to be realistic one must, however, take into account lags. Investment will not provide capacity production for some time to come. Either we must relate investment with output a year or two hence, or we may make a rough allowance by increasing the working capital in capital-supplying sectors. The latter amounts to assuming that the capital supplying sectors maintain the capital goods as inventories until they attain production[5]. For want of information such allowance must be

[4] *See* [13]. Pakistan's Third Plan approximately conforms to this assumption at the aggregate level; annual phasing of investment is such that total investment increases every year by about a thousand million rupees [10]. One could use other kind of stock-flow conversion factor for numerical extrapolation of investment by assuming that investment rises *smoothly* in some other manner from the base to the target year. To give an example, let

e^{5r} = investment index in the terminal year, with the annual exponential growth rate = r,

$\int_{t=0}^{5} e^{rt}dt$ = total investment index over the five years.

Then target year investment is the following proportion of total five-year investment

$e^{5r} \Big/ \int_{t=0}^{5} e^{rt}dt = r/(1 - e^{-5r})$ so that we have

$$dI_i^e = \left(\frac{r}{1 - e^{-5r}}\right) \sum_j b_{ij}^e X_j^e - I_{io}^e \text{ and so on.}$$

We use the stock-flow conversion factor on the basis of the assumed growth of investment of the type $I_t = I_o + Qt$ because, as just stated, the Third Plan (to which we apply the model) assumption roughly corresponds to this kind of hypothesis. For an example of the use of the hypothesis $I_t = e^{rt}$, *see* [6]. The basic assumption underlying the use of stock-flow conversion factor is that investment rises *smoothly* in some manner. One can easily think of situations in which this will not hold, *e.g.*, wartime mobilization, technological indivisibility, *etc.*

[5] The method is suggested in [7].

based on simple rules of thumb derived on the basis of limited enquiry in the field. If it is found that the lag is, say, one year, the working capital coefficient (s^e_{ii}, if we are considering the i-th capital-supplying sector) has to be increased by one. This would be similar to the assumption that investment of i sector's product in year o creates capacity output in year 1. We adopt this procedure for its great simplicity and assume the gestation lag to be *on the average* one year for engineering capital and one and half year for construction.

Substituting equation (viii) into equation (v) we obtain the following set of balance equations for region E; we have a similar set of balance equations for region W which can be obtained by appropriate changes in the superfixes.

$$\text{(ix)}\quad X^e_i = r^{ee}_i \sum_j (a^e_{ij} + 1/3k^e_{ij})X^e_j + r^{ew}_i \sum_j (a^w_{ij} + 1/3k^w_{ij})X^w_j + r^{ee}_i(dC^e_i + dG^e_i + dD^e_i + dE^e_i - dN^e_i - 5/3I'^e_{io}) + r^{ew}_i (dC^w_i + dG^w_i + dD^w_{ij} + dE^w_i - dN^w_i - 5/3I'^w_{io})$$

For reasons stated below, balance equations for services sectors would need adjustment. Thus the above equations describe the first twelve constraints of the model—one for each of the remaining six sectors in each of the two regions.

3.2 Adjustment in the Balance Equation for the Services Sectors

In the application of our model we measure all variables and coefficients, including the foreign and interregional trade entries, at purchasers' prices so that each industry is assumed to pay the trade and transport costs on all its sales of output, and the value of these services together form the trade and transport input into that industry.

There exists a discrepancy between the imports at purchasers' price and the foreign exchange cost of imports and the difference is accounted for by duty, tax and trade and transport inputs required to take the imported goods to their users within each region. This means that an expansion of imports from abroad or from the other region would require additional regional output of trade and transport services, the amount of which would be determined endogenously via the trade and transport input coefficient into imports. In the output equation for the services sector trade and transport input into imports must be accounted for.

$$X^e_s = \sum_j a^e_{sj} X^e_j + \sum_j T^{we}_j + \sum_j H^e_j + dC^e_s + dG^e_s$$

Where, the first term on the right is total intermediate demand for services;

T^{we}_j = region E's trade and transport output used as input required to take imports of j from W to various users in E;

H_j^e = trade and transport required similarly for imports from abroad and

dC_s^e and dG_s^e = consumption demand for services in East respectively by private and public sectors.

All the above elements are in terms of increases over the five-year period.

We assume that region E's trade and transport required per unit of j import from W into E is fixed

$$T_j^{we} = t_j^{we} X_j^{we}$$

whence we have, by using (iii)

$$T_j^{we} = t_j^{we} r_j^{we} Z_j^e$$

$$= t_j^{we}\ r_j^{we} \left[\sum_i \left(a_{ji}^e + 1/3k_{ji}^e \right) X_i^e \right] + t_j^{we}\ r_j^{we}\left(dC_j^e + dG_j^e + dE_j^e + dD_j^e - dN_j^e - 5/3I_{jo}^{\prime e}\right)$$

We similarly assume

$$H_j^e = h_j^e\ dN_j^e$$

Thus we have the output equation for services sector as linear function of all sectoral outputs and imports :

$$X_s^e = \sum_j a_{sj}^e X_j^e + \sum_j t_j^{we} r_j^{we} \sum_i \left(a_{ji}^e + 1/3k_{ji}^e \right) X_i^e + \sum_j t_j^{we} r_j^{we} \left(dC_j^e + dG_j^e + dE_j^e + dD_j^e - dN_j^e - 5/3I_{jo}^{\prime e}\right) + \sum_j h_j^e dN_j^e + dC_s^e + dG_s^e$$

We have a similar equation for X_s^w. These two, together with the twelve balance equations mentioned under 3.1, define the first fourteen constraints of the model. These 14 balance equations are shown in matrix form in Table XVI.

3.3 Other Constraints

The next set of constraints refers to private consumption. The minimum increase in total private consumption in each region may be specified as a target. Relative regional sizes of such targets may be based on the notion of the desirable reduction in the disparity of the regional standards of living[6]. The minimum

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[6] This has to be related to constraints 43 and 44 shown below. One may also introduce additional constraints to provide that the *relative* increases in the two regions' total consumption are not greatly dissimilar to the levels stipulated in setting the targets initially.

increase in private consumption demand of various sectors' products is given by this target via the expenditure elasticities of demand[7].

$$(15 - 20) \quad dC_i^e \geq e_i^e \left(C_{io}^e \Big/ \sum_i C_{io}^e \right) \sum_i \overline{dC_i^e}$$

$$(21 - 26) \quad dC_i^w \geq e_i^w \left(C_{io}^w \Big/ \sum_i C_{io}^w \right) \sum_i \overline{dC_i^w}$$

where e_i^e = expenditure elasticity of demand for i in East

$C_{io}^e \Big/ \sum_i C_{io}^e$ = base year consumption proportion of i in East.

In this particular application we define minimum necessary increase in national consumption to be 31.5 per cent (Third Plan target) and obtain its regional breakdown on the assumption that per capita consumption disparity would have to be reduced by a fifth (Third Plan objective about the reduction in regional inequality in terms of income). Subtracting public consumption we obtain minimum target increase in private consumption in each region—6,138.5 million rupees in East and 4,882.7 million rupees in West.

The next set of constraints refers to imports. Increase in total foreign exchange expenditure is limited by the sum of the increases in exports and foreign assistance.

$$(27) \quad \sum_j l_j^e \, dN_j^e + \sum_j l_j^w \, dN_j^w \leq \sum_j dE_j^e + \sum_j dE_j^w + \text{increase in foreign assistance.}$$

Foreign exchange cost per unit of import of j in East at purchasers' price is denoted by l_j^e.

Due to the special nature of the construction and fuel and power sectors, upper limits must also be specified for the import of their products. Imports of construction sector's products consist of cement and other construction materials and cannot exceed the current self-inputs of construction sector. Certain products of fuel and power sector are importables (*e.g.*, coal and oil)

[7] This follows from the definition of expenditure elasticity (e_i^e) which is given by

$$e_i^e = \frac{dC_i^e}{\sum_i dC_i^e} \cdot \frac{\sum C_{io}^e}{C_{io}^e} \quad \text{from which we have}$$

$$dC_i^e = e_i^e \left[C_{io}^e \Big/ \sum_i C_{io}^e \right] \sum_i dC_i^e$$

on the basis of which we define the constraints.

while others are non-importable (*e.g.*, electricity). We assume that incremental ratio of importable fuel to total fuel is the same as the average base-year (1964/65 which is the base-year of the Third Plan and also of our exercises) ratio. This does not appear unrealistic. What proportion of new vehicles and machines would use oil rather than some other kind of fuel can reasonably be estimated from the pattern of use at present. We assume that all fuels except electricity, natural gas and firewood are importables (.686 and .522 respectively in East and West of total fuel used in 1964/65).

(28) $dN_b^e \leqslant a_{bb}^e X_b^e$

(29) $dN_b^w \leqslant a_{bb}^w X_b^w$

(30) $dN_p^e \leqslant \beta_1 \left[\sum_j \left(a_{pj}^e + 1/3 k_{pj}^e \right) X_j^e + dC_p^e + dG_p^e + dE_p^e - 5/3 I_{po}^{\prime e} \right]$

(31) $dN_p^w \leqslant \beta_2 \left[\sum_j \left(a_{pj}^w + 1/3 k_{pj}^w \right) X_j^w + dC_p^w + dG_p^w + dE_p^w - 5/3 I_{po}^{\prime w} \right]$

where $\beta_1 = .686$ and $\beta_2 = .522$.

Minimum level of imports of engineering and fuel and power need not be specified because the model will automatically favour their import, capital income ratio being very high for these sectors. Import of agricultural products (unprocessed grains, *etc.*) under PL 480 is estimated by the Third Plan to go up by about 60 million rupees [10, pp. 94-95]. We assume that this aid cannot be converted into some other kind of aid so that increase in imports in terms of foreign exchange cost of agricultural goods must be at least 60 million[8].

(32) $l_a^e dN_a^e + l_a^w dN_a^w \geqslant 60.0$

Minimum import of food, construction material and other manufactures and minerals has to be specified as equal to the increase in the "non-competitive imports" of these goods. We do not have enough information to obtain precise estimates of such non-competitive imports. But in the base-year in Pakistan there were considerable amounts of competitive imports of the products of all these sectors so that it is a fair assumption that *increase* in imports of these products must at least be non-negative.

(33) $dN_f^e \geqslant 0$

(34) $dN_b^e \geqslant 0$

(35) $dN_r^e \geqslant 0$

(36) $dN_f^w \geqslant 0$

(37) $dN_b^w \geqslant 0$

(38) $dN_r^w \geqslant 0$

[8] We are also assuming (for want of information) that base-year non-PL 480 agricultural imports are non-competitive. This being a small quantity, the assumption is not so harmful.

The upper limit on investment can be specified either by putting a fixed ceiling $(I \leqslant \bar{I})$ or by putting an upper limit on the marginal rate of investment $\left[dI \leqslant \alpha \left(\sum_j {}^e_j X^e_j + \sum_j v^w_j X^w_j \right) \right]$. Since one of our purposes is to compare our exercises with the Third Plan, we assume that the upper limit of total gross investment over the Third Five-Year Plan period is given at the level stipulated by the Plan[9].

$$(39) \quad 52{,}000.0 \geqslant \sum_i \sum_j k^e_{ij} X^e_j + \sum_i \sum_j k^w_{ij} X^w_j + \sum_{t=1}^{5} \bar{D}^e_t + \sum_{t=1}^{5} \bar{D}^w_t$$

If some incremental self-sufficiency is not postulated for the engineering sector which supplies the entire non-construction capital goods, then the model would favour the satisfaction of the demand for this sector's products through imports (as far as permissible by the foreign exchange constraint) rather than by domestic production because of its very high capital requirement per unit of value-added. The Third Plan aims at attaining some self-sufficiency in this sector. One can discern a number of arguments in favour of this objective from the Third Plan itself: export earnings are unlikely to grow fast enough in future and are rather uncertain; foreign assistance is uncertain too and the perspective plan has the objective of being free of the necessity of foreign assistance and so on. It is therefore unwise to depend on imports for the entire supply of machinery for investment; domestic capacity for these goods should be expanded gradually[10].

The Third Plan aims at achieving a 45 per cent incremental self-sufficiency in capital goods. We, therefore, require that in *incremental* terms national production of engineering sector must be at least 45 per cent of national use of this sector's products. It would be desirable to specify certain degree of regional self-sufficiency in this sector as well for similar reasons, but such specification

[9] The rationale of this constraint is that unless we put an upper limit on total investment, there is nothing in the model which will prevent the terminal-year investment and income from being very high. This would mean that in the intervening years consumption would have to be curtailed to the required extent—perhaps below certain 'feasible' level. The specification of an upper limit on investment would be a safeguard against this. Note that this constraint puts an upper limit on the terminal capital stock.

The limit on investment in our model has to be set on the basis of independent considerations such as the desirable volume of terminal capital, capacity to mobilize domestic and foreign savings and judgments about the feasible maximum investment (minimum consumption) in the intervening years based perhaps on aggregative exercises.

[10] The Planning Commission puts forward an additional justification: "The past experience, particularly during the First Plan period, shows that it is fairly difficult to obtain a high rate of saving from the economy by concentrating on the production of consumption goods" [10, p. 34]. In the present model we simply assume that the Government is able to keep consumption within the specified limits and do not go into the problem of the administrative feasibility of alternative policies to restrain consumption. If it is found that capital-goods oriented industrialisation provides a higher rate of saving, then of course it is an additional justification for such policy. For an argument that this may actually be the case in Pakistan, *see* [11].

should generally be below the national level as long as some interregional trade is possible, so that regional comparative advantage criterion may have some role to play. The Third Plan does make the provision for the decentralisation of capital goods complex but the actual regional targets are not specified. We arbitrarily require minimum regional incremental self-sufficiency in engineering goods to be 30 per cent.

$$(40)\ X^e_m + X^w_m \geq .45 \Big[\sum_j \left(a^e_{mj} + {}^1/_3 k^e_{mj}\right) X^e_j + \sum_j \left(a^w_{mj} + {}^1/_3 k^w_{mj}\right) X^w_j + dC^e_m + dC^w_m + dG^e_m + dG^w_m + dE^e_m + dE^w_m + dD^e_m + dD^w_m - {}^5/_3\left(I'^e_{mo} + I'^w_{mo}\right)\Big]$$

$$(41)\ X^e_m \geq .30 \Big[\sum_j \left(a^e_{mj} + {}^1/_3 k^e_{mj}\right) X^e_j + dC^e_m + dG^e_m + dE^e_m + dD^e_m - {}^5/_3 I'^e_{mo}\Big]$$

$$(42)\ X^w_m \geq .30 \Big[\sum_j \left(a^w_{mi} + {}^1/_3 k^w_{mj}\right) X^w_j + dC^w_m + dG^w_m + dE^w_m + dD^w_m - {}^5/_3 I'^w_{mo}\Big]$$

The Third Plan sets out the objective of reducing the current interregional disparity in per capita output by about a fifth. There are reasons to believe that this is the minimum reduction in disparity which is aimed at. We should also introduce a constraint showing the maximum desirable reduction in disparity. This is necessary because a too rapid reduction in disparity may mean a demoralisingly low rate of growth for West and hence be politically and socially undesirable. We stipulate that the maximum permissible reduction in inequality in per capita income would be by a third.

$$(43)\ 1.069 \sum_j v^e_j X^e_j - \sum_j v^w_j X^w_j \geq 1{,}195.6$$ [11]

$$(44)\ 1.03 \sum_j v^e_j X^e_j - \sum_j v^w_j X^w_j \leq 2{,}002.6$$

[11] We have base-year inequality $(Y^w_o/H^w_o)/(Y^e_o/H^e_o) = 1.344$ (Y^e = regional product in East, H^e = population in East, similarly Y^w and H^w, subscript = time period). We must have $(Y^w_t/H^w_t)/(Y^e_t/H^e_t) \leq 1.275$ if disparity has to be reduced by at least a fifth. But H^w_t/H^e_t = 58.6/69.9, so that $Y^w_t/Y^e_t \leq 1.069$ from which we obtain $1.069 \triangle Y^e \geq \triangle Y^w + Y^w_o - 1.069\ Y^e_o$. Substituting the values of Y^w_o and Y^e_o we get (43). Similarly the condition that disparity in per capita income should not be reduced by more than a third gives (44).

Why not specify limits of incremental inequality in terms of consumption rather than income so that production may be located purely on efficiency considerations and distribution may be taken care of by interregional income transfers? The answer seems to be the following. It is unlikely that such income transfer would be feasible for any reasonable length of time both politically and from the standpoint of fiscal policy. The poorer region may not like it even for a short period because such policy would limit its future capacity for growth.

Government consumption, exports, foreign assistance and replacement are exogenously given. For the first three we use Third Plan figures (disaggregated where necessary on the basis of relevant complementary information); replacement estimates are based on the trend of gross investment in the past and assumed life tables for various kinds of capital equipment.

3.4 Constraints on the Agricultural Sector

The treatment of agriculture, for its vastness and peculiarities, presents problems. First, one has to decide about the type of relationship to use. Is it justified to assume for the agricultural sector the standard input-output relation which allows for no substitution among inputs? Certainly, the assumption is too strong; agricultural inputs do not always seem to stand in any fixed relation to one another and to agricultural output, in the way inputs into manufacturing activities seem to do.

Is it then justified to assume a production function of the perfect-substitutability among inputs type? Here too, one faces formidable difficulties. First, although substitution is perhaps possible over a *considerable* range between *some* factors, the assumption of unlimited *perfect* substitutability among *all* factors is as implausible as that of *perfect* complementarity. Secondly, the actual estimation of the production functions with the property of substitution requires a great deal of experimental data and not just the picture of the economy at one or a few points of time. For Pakistan we do not have any such information.

The more general production functions which suffer less from the above criticisms cannot be used either because they cannot be incorporated in the simple input-output linear programming framework or because even their very crude statistical estimation requires much more information than is available.

Considering the above we decide to stick to the usual fixed coefficients assumption and try to give it some amount of formal justification by the following kind of reasoning: agriculture may be presumed to have a number of alternative processes. Once a process (presumably the best) is chosen, it can be described by a set of fixed input coefficients.

As the use of some factor inputs increase more than others, there would almost certainly be diminishing returns. More important, as agricultural expansion is accelerated, there is the extreme likelihood of diminishing returns to *all* (or most) factor inputs and of the "returns to scale" to diminish more, the faster the rate at which agriculture is being driven. The reason the assumption of this kind of "diminishing returns to scale" seems plausible is that in the production function we are not able to include all factors which contribute to output. Notable omissions are the supply of land of unchanged average quality and the adaptation of the farmers to changed pattern of factor use (which takes time). These factors are unlikely to increase in proportion to other factor inputs. To reflect the difficulty of increasing the agricultural output at the margin, we use marginal coefficients for most inputs into agriculture higher than the corresponding average coefficients.

Finally the extent of "diminishing returns to scale" in agriculture is likely to depend on the rate of expansion that is being attempted. If agriculture is being driven at 10 per cent per year the marginal coefficients would almost certainly have to be higher than if agriculture is being driven at 5 per cent per year. Thus with respect to each set of marginal coefficients it is sensible to specify an upper limit on agricultural expansion. In our example we stipulate maximum possible rates of expansion respectively at 6.5 and 6.0 per cent for East and West Pakistan[12]. The higher rate for East is justified by the widely prevalent belief that it is easier to obtain an increase in agricultural output in East than in West.

$$(45) \quad X_a^e \leqslant (1.065)^5 \, \bar{X}_{ao}^e - \bar{X}_{ao}^e$$

$$(46) \quad X_a^w \leqslant (1.060)^5 \, \bar{X}_{ao}^w - \bar{X}_{ao}^w$$

where $\bar{X}_{ao}^e$ and $\bar{X}_{ao}^w$ are benchmark base year figures (*not* increases in the base year).

It should be emphasized that the incremental capital coefficients for agriculture, though assumed higher than average, insufficiently reflect the difficulty of expanding this sector. Ensuring domestic production and import of inputs would not be enough, their application would require great effort on the part of the Government through the provision of extension services and credit facilities. In our model extension services are included under public expenditure whose composition must undergo appropriate change.

4. THE OBJECTIVE FUNCTION

The main objectives of the Third Five-Year Plan are increasing national product, reducing regional disparity in income, achieving certain targets with respect to employment and export, attaining a certain degree of self-sufficiency in the production of capital goods and certain "social objectives such as diminishing inequalities in the distribution of income, wealth and economic power" [10, pp. 39-40].

In the following exercises, the objective the optimizing models set out to maximize is a function of the regional products. Certain other objectives are however

[12] This means that at the national level we are assuming the maximum possible rate of growth of agriculture to be about 1.25 percentage points higher than the planned rate of growth postulated by the Third Plan.

taken into account by the introduction of appropriate constraints: removal of disparity in regional incomes by a certain amount and the achievement of a certain degree of self-sufficiency in the production of capital goods are provided for in this way. The two objectives we cannot take into account are employment and income distribution. These two are essentially the same, however. Increasing employment even at the cost of some material output could conceivably be desirable for its distributional effects only. The neglect of these factors amounts to the assumption that the desirable income distribution may be attained by other means such as taxation and Government expenditure, the specific measures relating which may be planned without reference to the details of the present model. This assumption is not entirely satisfactory but is made because there is no simple way of incorporating the problem of income distribution into the present model.

Thus our objective is to maximize 'utility' or 'welfare' which is a function of national output and its distribution between the regions. The question of interregional distribution can be introduced in one of the two ways. The first is to define the objective function as the simple sum of regional products and define the permissible upper and lower limits of incremental inequality as in the constraints (43) and (44) above. Alternatively the objective function could be defined as the weighted sum of regional products ($U = L \sum_j v^e_j X^e_j + (1 - L) \sum_j v^w_j X^w_j$ where the weights L and (1 — L) are proportional to 'marginal utilities' attached to the regional outputs) and it could be maximized subject to the usual constraints but *without* the interregional constraints of the type (43) and (44).

Let us illustrate by referring to Figure 1. EE′ is the production possibility curve defined by the constraints (1) to (42) and (45) to (46) (but *not* the interregional constraints that (43) and (44) denote) which indicates various efficient combinations of Y^e and Y^w. If the objective is to maximize the simple sum of the regional products, the maximand is the straight line VV′ with slope − 1 and the optimum solution would be given by A, the point of tangency between VV′ and EE′.

This solution however does not take into account the important consideration that there may be a (presumably political) limit of interregional inequality and/or that the planners may want to attach greater importance to the generation of output in the poorer region.

One way to introduce the question of the interregional distribution of incremental output is to introduce constraints of the type shown above in (43) and (44) defining a lower and an upper limit on the incremental income disparity between the regions. In Figure 1 these constraints are denoted by the straight

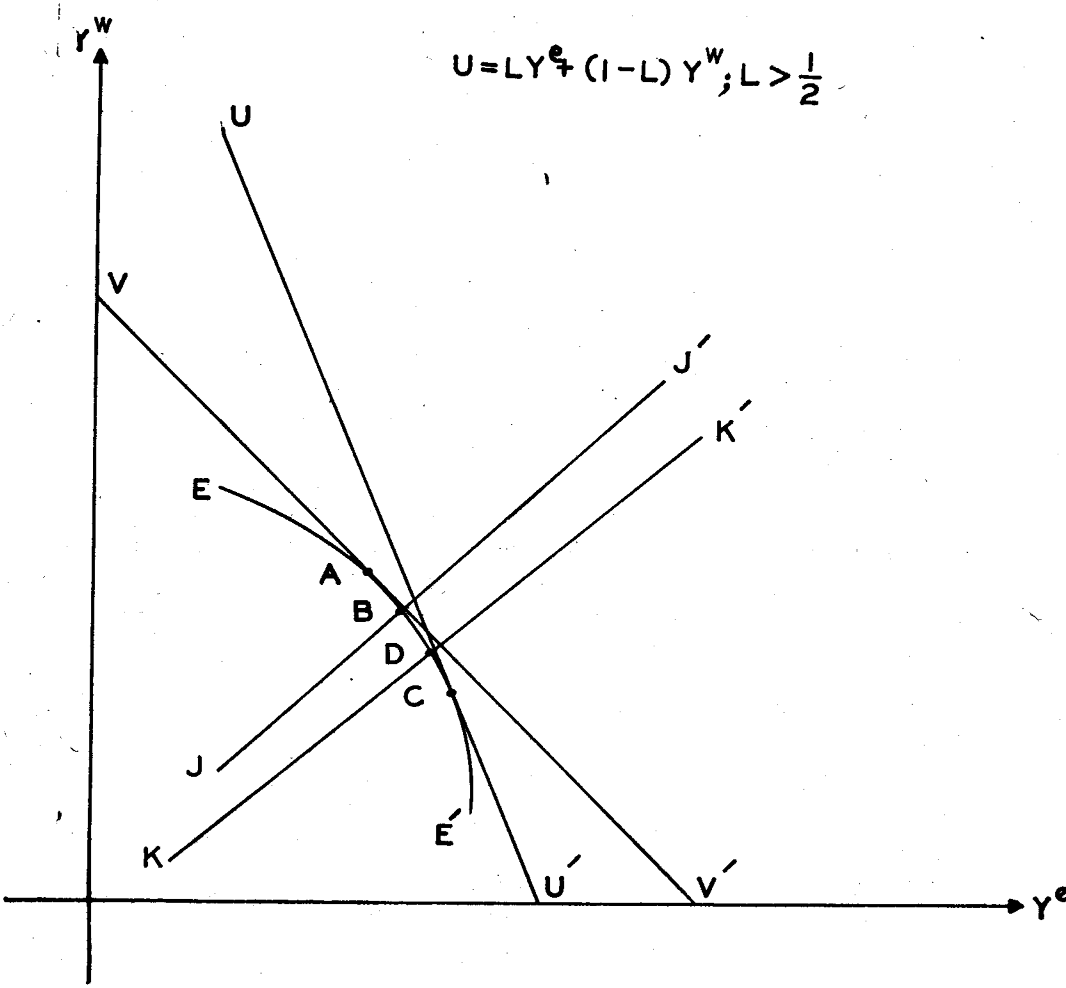


Figure 1

lines JJ′ and KK′. The new optimum must be within that part of the production possibility curve which lies between the lines JJ′ and KK′. If the original optimum A were strictly within this region, then the constraints JJ′ and KK′ would be irrelevant (*i.e.*, non-binding), otherwise the new optimum would lie on one of the lines defining the interregional constraints. In Figure 1 it is at the point B, that of intersection between JJ′ and EE′.

The other way of introducing the question of interregional distribution is to use an interregional welfare function. If a linear welfare function is used which attaches a higher weight ("marginal utility") to the output of the poorer region (which is East), the maximand would be of the shape indicated by the line UU′, steeper than VV′. If we want the extent of interregional disparity to be determined by the production possibility surface and the marginal utilities of the regional incomes alone, the constraints showing the permissible range of interregional disparity (JJ′ and KK′) must be removed. This is because the point of tangency between UU′ and EE′ may lie outside the range between JJ′ and KK′ (as it does at point C in the figure) in which case the new optimum without the interregional constraints (point C) and the new optimum with the interregional constraints (point D) would be different.

Note that the points A, B and C are on the efficiency frontier; by varying the values of the weights assigned to the regional products in the interregional welfare function, additional points on the production possibility curve may be obtained. This would enable us to discover the approximate shape of the production possibility frontier in the relevant range.

The final choice of a plan would still require a decision about the 'desirable weights', *i.e.*, a 'desirable' ratio of marginal utilities of regional incomes, but once the shape of the efficiency frontier is known, such choice is made a great deal easier. This is because the efficiency locus would tell what 'sacrifice' in terms of national product is associated with any given change in the interregional welfare function (*i.e.*, in the slope of the line UU′ in the figure). The decision-making authority will be able to use this information in arriving at their choice.

The following are the maximizing exercises we carry out :

1) The basic optimizing model (BOM) maximizes the sum of the regional products subject to the constraint that the interregional disparity in terms of per capita output must be removed by at least a fifth and at most a third :

$$\text{Max } U = \sum_j v_j^e X_j^{\prime e} + \sum_j v_j^w X_j^w$$

2) The welfare maximizing model (W-max model) maximizes an interregional welfare function in which the marginal utility of East's regional product is 20

per cent higher than the marginal utility attached to West's regional product[13] subject to no other constraint that specifies a predetermined amount by which the interregional disparity must be removed:

$$\text{Max } U = L\sum_j v_j^e X_j^e + (1-L)\sum_j v_j^w X_j^w \text{ where } L/(1-L) = 1.2,$$

$$\text{or} \quad \text{Max } U = .545\sum_j v_j^e X_j^e + .455\sum_j v_j^w X_j^w$$

3) The GDP maximizing model maximizes the simple sum of the regional products subject to *no* other constraint that specifies a predetermined amount by which the interregional disparity must be removed. The maximand is the same as in the BOM but the BOM constraints showing the permissible range of reduction in interregional inequality are removed from the GDP maximizing model.

4) The Third Plan allocation model ("TFYP model") maximizes the sum of the regional products with the Third Plan regional allocation of investment given as independent constraints but with *no* other constraint about the permissible range of reduction in interregional inequality. Thus the maximand is the same as in the BOM but there are the following differences between the two models regarding the constraints subject to which the objective is being maximized : *a*) the BOM constraint on the upper limit on total *national* investment is replaced by two separate constraints showing investment in each region at the levels postulated by the Third Five-Year Plan ($I^e = 27{,}000$ and $I^w = 25{,}000$), and *b*) the BOM constraints on the permissible range of reduction in interregional disparity are removed from the TFYP model because the reduction in disparity would not follow from the given regional allocation of investment.

In terms of Figure 1, the BOM represents the point B, the W-max model the point C and the GDP maximizing model the point A. Note that all these three are points on the efficiency frontier. The relative position of these three points would give an indication of the shape of the production possibility curve in the

[13] The final choice of the value of L seems to be a matter of political decision and its possible range will depend on the political machinery which is being used to determine it. A value of L much in excess of 0.5 would have little chance of being accepted by the present national legislature (in which the regions have equal representation) but would have a much better chance of beuuıgi accepted a national referendum based on universal suffrage (if only because Easterners are more numerous). The machinery to decide on the value of L has to be devised after taking into account various political constraints. In general a value of L greatly different from 0.5 can not be obtained without weakening political goals such as greater national integration. The decision about the value of L has to be revised from time to time; once regional per capita levels of living get closer, there would be less justification to have a value of L much different from 0.5. At the moment the highest political authorities in Pakistan seem to have reached the general consensus of having an L greater than 0.5. The higher marginal utility attached to East's income is justified by the lower initial per capita income there. It is worthwhile to experiment with alternative values of L because by doing so one makes the problem of selecting an L easier by spelling out the implications of alternative Ls. More on this later.

relevant range. It is not known whether the TFYP model is also an efficient point on the same frontier because in this model two new constraints have been introduced.

5. THE DATA

The model puts considerable demand for statistical data. For its application, the model requires information about the following :

a) Incremental current input coefficients (the a^e_{ij} s and a^w_{ij} s) matrices for the two regions;

b) Incremental fixed and working capital coefficients (b^e_{ij}, b^w_{ij}, s^e_{ij} and s^w_{ij}) matrices for the two regions;

c) Interregional trade/supply ratios;

d) Base-year magnitudes about sectoral outputs and components of final demand;

e) Expenditure elasticities of demand;

f) Trade and transport input coefficients into imports and foreign-exchange cost coefficient of imported goods ; and

g) Replacement demand for various kinds of capital.

The methodology followed in obtaining these coefficients is too long to describe in this article. This is set out in details in [4, Chapter 4 and statistical appendix to Chapter 4]. Tables I to VIII show these coefficients and data.

6. SOLUTIONS OF THE OPTIMIZING EXERCISES AND THEIR COMPARISON

Tables XI to XIV summarise the solutions of the four optimizing exercises. It is of some interest to compare the four solutions with respect to some of their broad features.

The first important finding is that the BOM reduces the interregional inequality in per capita output by a fifth. This means that in the BOM the constraint showing the lower limit on the permissible range of reduction in interregional inequality (constraint (43)) is binding. The GDP maximizing model is the reformulation of the BOM without the constraints on incremental interregional disparity. Its solution indicates an aggravation of interregional inequality by the end of the Third Plan. To speak in terms of Figure 1, the solution of the BOM lies on the line JJ′. If the constraint (43) (denoted by JJ′) is removed, the solution moves somewhere in the direction of A—northwest of the solution of the BOM.

The reduction of inequality by a fifth means that East has about 43 per cent of gross investment (in the BOM). Thus we are faced with the finding that if we set out to maximize GDP without introducing constraints which secure a minimum allocation of investment to East so as to reduce the interregional inequality by at least a predetermined amount, the model does not favour East in the allocation of investment. How do we reconcile this with the fact that the overall incremental capital-income ratio is considerably lower in East than in West?

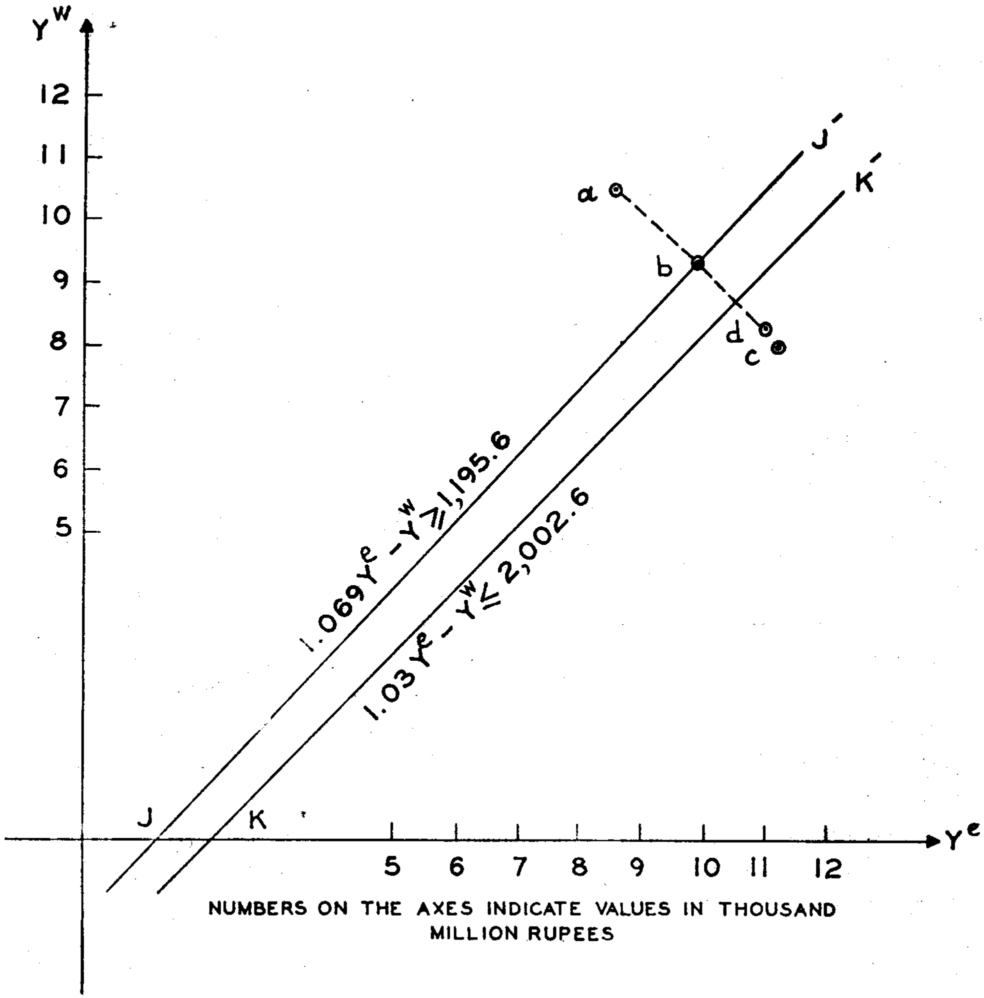
A verbal explanation may be attempted by advancing the following kind of reasoning: lower capital-income ratio in East derives mainly from East's lower ratio for agriculture. For most other sectors, East's ratios are considerably higher than West's (Table X). Agriculture, however, has got an upper limit in each region beyond which it cannot be driven. Once this limit is hit, it no longer remains advantageous to allocate more investment to East.

The next important finding refers to the value of the national product in the various maximizing exercises. As can be seen from the tables showing the four solutions, interregional distribution of the incremental GDP varies greatly between the maximizing models. Yet the GDP remains pretty much the same for all these solutions. Plotting the first three solutions (the BOM, the W-max model and the GDP maximizing model) on Figure 2, one can obtain the approximate shape of the production possibility curve in the relevant range[14]. Throughout the known range, the slope of curve does not deviate much from —1. In other words, the rate of transformation between the regional products is approximately unity in the known range. One expects the slope of the production possibility line to be like this around the GDP maximizing point. But here this slope obtains over a rather wide range. In other words, GDP maximizing points are spread over a relatively wide range. No simple verbal explanation of the phenomenon suggests itself, but one is led to conclude that even when the agricultural ceiling is hit in East, relative advantage of West over East must be very slight indeed.

The upshot of the above is that for fairly small changes in objectives one can obtain plans with greatly different regional shares of investment and incremental output. Thus by changing the value of L from 0.5 (in GDP-max model) to 0.545 (in W-max model), East's share of gross investment can be increased by about 41 per cent. Even with virtually unchanged objectives in terms of GDP one could justify widely varying plans. From the point of view of maximizing GDP it involves no appreciable gain or loss if one moves from the BOM (point b on Figure 2) to the point of unconstrained maximization of GDP (point a) or to the W-max point (d).

The fact that over a rather wide range of the production possibility curve the value of GDP is almost the same does not mean that all these points are equally

[14] The TFYP model is shown on the figure by the point C. As already stated, it may not be on the efficiency frontier, but does not seem to be much inside.



Y^W
12
11
10
9
8
7
6
5
a
b
d
c
J
K
J′
K′
$1.069Y^e - Y^w \geq 1,195.6$
$1.03Y^e - Y^w \leq 2,002.6$
5 6 7 8 9 10 11 12
Y^e
NUMBERS ON THE AXES INDICATE VALUES IN THOUSAND MILLION RUPEES

Figure 2

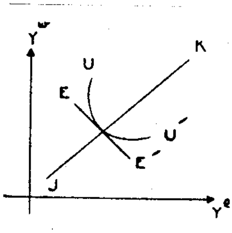
desirable. The slope of the production possibility curve indicates the rate of transformation, but the choice of an optimum point is dictated by the equality of the rate of transformation and the rate of substitution (the slope of the planners' "social indifference curve" indicating the relative valuations of the regional products at different levels of incremental interregional inequality)[15].

Thus once the efficiency frontier in the relevant range is known, the planners, in order to decide whether a particular point on the efficiency locus represents a "desirable" plan, must ask whether the transformation rate at the point also indicates a sensible rate of substitution between (*i.e.*, a sensible ratio of the marginal utilities of) the regional products and whether the point lies within the political tolerance limits.

Thus, one could argue that the point a is irrelevant to the planners. Compared to the BOM, it indicates a much worse interregional distribution of income (leading to the further *relative* impoverishment of the poorer region) without any corresponding gain in total GDP. If such a worse interregional distribution is to be accepted by the planners, the only justification must be considerable gain in terms of total GDP. In other words, at the level of incremental inequality denoted by the point a, the rate of substitution shown by the planners' "social indifference curve" (*i.e.*, U^e/U^w, where U^e=marginal utility of East's income *etc.*) would be considerably greater than 1 whereas the rate of transformation (in the direction of the BOM) would be 1.

On the basis of similar logic, a movement from the BOM in the direction of the W-maximizing solution (point d) would be desirable unless such movement is in excess of the permissible political limits because it would provide more to the poorer region without any appreciable reduction in total GDP. In the BOM such limit has been *arbitrarily* defined as the reduction in inequality in per capita output by a third, the point of intersection between KK′ and the efficiency frontier.

Although shifting of resources between regions provides no significant change in GDP in the relevant range, re-allocation of resources among sectors does provide marked change in output. This can be shown by comparing the maximizing exercises with the Third Plan (keeping in mind that the latter is not strictly



[15] Note that the "social indifference curve" is defined only within the range between the constraints showing the political tolerance limits of incremental inequality. Note also that the uniqueness of the optimum would require that either the rate of substitution is —1 only along a very narrow range and significantly different from —1 elsewhere (like UU′ in the adjacent figure) or that the political tolerance limit is very narrow (the limiting case being the complete overtapping of the upper and lower boundaries of incremental inequality like the line JK).

comparable with our exercises). All the optimizing exercises give significantly more GDP from no greater investment as compared to the Third Plan. For example, the BOM gives about 44 per cent increase in GDP compared to 38 per cent provided in the Third Plan. This can be largely explained by the fact that the optimizing exercises provide a more efficient allocation of investment and foreign exchange resources among sectors under the assumptions we make[16].

Let us broadly indicate the main differences between sectoral allocation provided by the optimizing exercises and those provided by the Third Plan. East's agriculture in all the optimizing exercises grows at the rate indicated by the constraint on the upper limit. West's agriculture grows at the maximum possible rate in the BOM and GDP-max models and slightly slower in the other two exercises. Agriculture claims just over a quarter of gross investment compared to just under 24 per cent proposed by the Third Plan[17].

Table XV shows the broad pattern of investment allocation among the non-agricultural sectors in some of our optimizing exercises and in the Third Plan. The Third Plan investments are gross while ours are net. We show in Table VIII total replacement for each kind of capital and not the replacement demand for each sector. We can of course roughly estimate the latter following the same method and using the same kind of information. But we avoid this additional work because we can make our point without this. If, however, we assume that the share of replacement in gross investment is same for all non-agricultural sectors (which is unlikely to give greatly inaccurate figures because total replacement in these sectors is a fairly small proportion of total investment because of the fact that much of the capital stock is "young"), then gross sectoral investments can be found by stepping up net investments by about 5 per cent.

The optimizing exercises indicate much smaller allocation of investment to fuel and power and other manufacturing (and possibly to food processing as well) and considerably more to services, construction and engineering as compared to the Third Plan. A particularly noticeable feature of the optimizing

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[16] The smaller GDP in the Third Plan as compared to the optimizing exercises could conceivably be *partly* explained by the possible use of a set of capital coefficients different from ours. The reason we think that this is not the whole explanation is that we tried to "solve" the Third Plan (as far as we could find out its targets and assumptions) by using the first 14 balance equations and found that the *overall* regional capital coefficients were almost exactly the same as those assumed by the Plan. *See* [4, Chapter 5].

[17] The Third Plan allocates 15.4 per cent of investment to "agriculture". But the Plan concept of agricultural investment is rather curious: investment in irrigation is shown under water and power sector while certain costs relating to fertilizers are regarded as capital inputs in agriculture. For the last point, *see* [9, Chapter 1]. In the past about 60 per cent of investment in water and power was found to be investment in agriculture (*see* [1]). Using this ratio and assuming that about 5 per cent of the investment shown in the Plan under agriculture consists of cost relating to the distribution of fertilizer, we find that the Third Plan allocation to agriculture is about 23.7 per cent of total gross investment.

exercises is the spectacularly fast rate at which construction sector has to grow. In the BOM the construction sector has to grow by 138 per cent over the five-year period for Pakistan as a whole (which is not much different from the rate at which the other optimizing models want it to grow). Much of it is caused by investment demand by other sectors and demonstrates how high a proportion of investment consists of construction[18].

A comparison of the solutions indicates that the reduction in interregional inequality would require smaller effort in terms of East's share of total investment than is supposed by the Plan. The BOM reduces the interregional inequality by a fifth (which is the Third Plan objective) but requires the allocation of only about 43 per cent of investment to East compared to the Third Plan allocation of 52 per cent. Using the Third Plan allocation in our framework we find that the interregional disparity is reduced by nearly a half—much more than a fifth which is the Third Plan objective.

This comparative ease in securing output increase in East derives largely from the fact that the ceiling in agricultural output has been set at a relatively high level and that East's agriculture needs least capital to expand. But although its capital requirement is low, such a big agricultural programme would require organizational reform. Over a five-year period extensive land reform measures are unlikely to produce effective results. So the main burden has to be borne by other organizational reforms. Ensuring domestic production and import of inputs would not be enough. Their application in a small peasant ownership dominated agriculture would require great effort on the part of the Government through the provision of credit and extension services. If it is found necessary to lower the ceiling on East's agricultural expansion, the relative ease of securing a given reduction in interregional inequality would gradually disappear.

In the BOM, the marginal saving rate would have to be lower in East (0.25) than in West (0.30) while at the national level it would have to be 27 per cent—slightly higher than 26 per cent postulated by the Third Plan. In the W-max and TFYP models, incremental saving rates in East (0.32 and 0.37 respectively) have to be considerably higher than in West (0·25 and 0·22). Except in the GDP-max model, East would require a very great acceleration of investment (from 11 per cent of regional output in the base year to 19.5 per cent in the BOM and about 25 per cent in the TFYP model in the terminal year) while in West it requires relatively small acceleration (from 18 per cent of regional output in the base year to 21.6 per cent in the BOM and 18 per cent in the TFYP model in the terminal year). Given the well known fact that private investment propensity is

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[18] This is something Reddaway notices for India [12, p. 98].

much higher in West, this means that a large share of public investment has to be allocated to East[19].

Another important feature of the optimizing exercises is that they all favour the allocation of a bigger share of foreign exchange to East Pakistan. Imports are favoured in those sectors in which domestic production uses up relatively more capital. Such sectors are mainly fuel and power, engineering and other manufactures, services being non-importable. For these sectors East's capital income ratios are more often greater than West's.

7. SOME CONCLUDING REMARKS

In view of the results presented above, it seems justified to claim that the model is capable of analysing some of the planning problems in a multi-region economy. Once the planners in such an economy have defined their objective or welfare function and made decisions about exogenous factors such as the maximum rate of investment and the availability of foreign exchange and the provision of minimum consumption, the model provides the allocation of investment and foreign exchange between the regions and among the sectors within each region so as to maximize welfare.

It is easily recognized that the present model indicates the allocation of resources only among the broad sectors and regions whereas practical planning must concern itself with many detailed features of the economy. Planners, however, may find it advantageous to *plan in stages* (*i.e.*, first determine some of the most important variables and later determine others relating to the detailed features) particularly if the state of statistical information and the capacity to handle computational burden are undeveloped (as they certainly are in Pakistan). This in effect is the well-known scientific method of successive approximations. The main advantage of this method is the economy of effort and the ability to produce a solution where a better and more general method of producing a solution does not yet exist.

The solutions of the exercises of this kind are numerically sensitive to the estimates of the structural coefficients. In the present application, the estimated coefficients are far from satisfactory. But we do not think that the numerical exercises are purely illustrative. This is because the estimated coefficients are believed at least to indicate the order of magnitudes of the structural relationships they represent. Secondly, if the errors in estimation are random (as we hope they

[19] East has indeed a bigger share of the public sector investment in the Plan, but the present allocation presupposes that private investment in the two regions would be equal. For this, considerable preferential treatment to the private sector in East would almost certainly be necessary.

are), then with our highly aggregated sector classification some kind of averaging of errors should leave the estimates relatively free of any systematic bias.

We do not claim that the results will be quantitatively very accurate, nor do we know the probable extent of error. But the solutions can perhaps serve as broad guides for the allocation of investment and foreign exchange. In general it may be advisable for the Plan executing authority to adhere to these broad limits and require that a special case must be made by pointing out the invalidity of one or more assumptions on which the exercise is based if these limits are to be exceeded.

Some results of the numerical exercises are of course interesting and instructive as qualitative deductions even if the actual magnitudes of such deductions are not considered to be very accurate. To illustrate, let us consider the following : the current input coefficients matrices and the expenditure elasticities clearly show that East Pakistan's technology and consumption demand are more agriculture-oriented than West's. As a direct consequence of this, the model indicates that (from purely feasibility consideration) if East is to grow faster than West then East's agriculture must grow even faster than West's. From the standpoint of optimality considerations the conclusion is reinforced by the fact that it is easier to secure increase in agricultural output in East than in West in terms of capital input requirement. That it is easy to forget such fundamental qualitative facts in planning is manifested by the Third Plan decision to drive West's agriculture faster than East's while at the same time having the objective of securing a higher rate of overall growth in East than in West.

The use of models in analyzing any actual planning process necessarily involves a certain degree of abstraction. As more and better data become available and the capacity to handle computational burden is increased, it should be gradually possible to reduce the degree of abstraction and make closer approximation of the planning process.

REFERENCES

1. Bergan, Asbjorn and Wouter Tims, *Capital/Output Ratios Related to Investments 1959/60—63/64.* Mimeographed, (Karachi: Planning Commission, Harvard Advisory Group, 1965).
2. Chenery, Hollis B., "Development Policy for Southern Italy", *Quarterly Journal of Economics*, 1962.
3. Chenery, Hollis B., Paul Clark and Vera Cao-Pinna, *The Structure and Growth of the Italian Economy.* (Rome: Mutual Security Agency, 1953).
4. Khan, Azizur Rahman, *Planning and Regional Development*; *The Application of Multi-sector Programming Model to Interregional Planning in Pakistan.* Unpublished Ph. D. thesis (University of Cambridge, 1966).

5. Leontief W. and others, *Studies in the Structure of the American Economy*. (London : Oxford University Press, 1953).
6. Manne, Alan and Ashok Rudra, "A Consistency Model For India's Fourth Five-Year Plan", *Sankhya*, Series B, 1965.
7. Mirrlees, J. A., *A Demonstration Model.* Unpublished manuscript. (New Delhi, 1963).
8. Moses, Leon, "The Stability of Interregional Trading Patterns and Input-Output Analysis", *American Economic Review*, 1955.
9. Pakistan, Planning Commission, *The Second Five-Year Plan (1960-65)*. (Karachi : Manager of Publications, 1960).
10. Pakistan, Planning Commission, *The Third Five-Year Plan (1965-70)*. (Karachi : Manager of Publications, 1965).
11. Power, J. H., "Industrialisation in Pakistan : A Case of Frustrated Take-Off?" *Pakistan Development Review*, Vol. III, No. 2, 1963.
12. Reddaway, W. B., *The Development of the Indian Economy*. (London : Allen and Unwin, 1962).
13. Sandee, Jan, *Demonstration Planning Model For India*. (London: Asia Publishing House, 1960).

TABLE I

INCREMENTAL CURRENT-INPUT COEFFICIENTS FOR THE THIRD FIVE-YEAR PLAN

Supplying sectors ↓ / Using→ sectors	A	F	M	B	P	R	S
			A. East Pakistan				
A	.179	.736	.221	.155	—	.256	—
F	.004	.001	—	—	—	.001	—
M	.005	—	.221	.046	.058	.028	.005
B	—	—	—	.106	—	—	.046
P	—	.002	.022	.004	.297	.019	.021
R	.024	.005	.022	.111	.017	.270	.023
S	.032	.069	.049	.013	.042	.144	.062
			B. West Pakistan				
A	.126	.653	.105	.025	—	.162	—
F	.024	.009	—	—	—	.003	—
M	.014	—	.316	.072	.040	.011	.010
B	—	—	—	.183	—	—	.011
P	—	.007	.027	.018	.163	.020	.067
R	.048	.054	.063	.204	.020	.320	.024
S	.045	.090	.078	.054	.186	.251	.044

TABLE II

INCREMENTAL FIXED-CAPITAL COEFFICIENTS

Supplying sectors ↓ / Using→ esctors	A	F	M	B	P	R	S
			A. East Pakistan				
M	.051	.120	.298	.068	2.794	.335	.506
B	.516	.044	.222	.008	2.752	.213	2.739
			B. West Pakistan				
M	.136	.119	.357	.115	2.525	.319	.562
B	1.220	.058	.192	.030	1.760	.131	1.835

TABLE III

INCREMENTAL WORKING-CAPITAL COEFFICIENTS

	A	F	M	B	P	R	S
			A. East Pakistan				
A	.135	.050	.159	.034	—	.056	—
F	.002	.204	—	—	—	—	—
M	—	—	2.079	.010	.023	.006	—
B	—	—	—	.968	—	—	.002
P	—	—	.015	.001	.390	.004	.001
R	—	—	.015	.024	.007	.364	.001
S	—	—	—	—	—	—	—
			B. West Pakistan				
A	.069	.079	.080	.005	—	.036	—
F	.005	.144	—	—	—	.001	—
M	—	—	1.614	.015	.023	.002	.001
B	—	—	—	1.193	—	—	.001
P	—	.001	.017	.004	.200	.004	.006
R	—	.007	.041	.043	.012	.330	.002
S	—	—	—	—	—	—	—

TABLE IV

INTERREGIONAL TRADE SUPPLY RATIOS FOR 1964/65

i	r_i^{ee}	r_i^{we}	r_i^{ew}	r_i^{ww}
A	.988	.012	.022	.978
F	.982	.018	.002	.998
M	.820	.180	.006	.994
B	1.000	—	—	1.000
P	.982	.018	—	1.000
R	.805	.195	.050	.950
S	1.000	—	—	1.000

TABLE V

EXPENDITURE ELASTICITIES AND MARGINAL CONSUMPTION COEFFICIENTS (M.C.C.)

	East Pakistan		West Pakistan	
	Expenditure elasticity	M.C.C.	Expenditure elasticity	M.C.C.
A	1.25	.358	1.29	.393
F	.65	.334	.58	.229
M	1.81	.005	1.25	.014
P	1.25	.010	1.25	.028
R	1.81	.163	1.25	.199
S	1.31	.130	1.26	.137

TABLE VI

i	t_i^{we}	t_i^{ew}	h_i^e	h_i^w
A	.295	.166	.251	.185
F	.362	.098	.361	.096
M	.048	.048	.071	.068
B	—	—	.200	.200
P	.313	—	.448	.294
R	.231	.231	.159	.159

TABLE VII-A

POSSIBLE STATE OF AFFAIRS IN 1964/65 IN EAST PAKISTAN

(*in million rupees : current purchasers' price*)

	A	F	M	B	P	R	S	$\sum_j X_{ij}$	Private consumption	Public consumption	Fixed investment	Working capital increase	Replacement	Exports	Imports	Trade & transport into imports
A	2,819.8	6,599.1	79.8	261.7	—	735.4	—	10,495.8	5,097.3	—	—	162.4	—	326.8	329.5	...
F	63.0	9.0	—	—	—	2.9	—	74.9	9,171.4	91.7	—	63.7	—	31.3	466.8	...
M	78.8	—	66.5	77.7	6.0	80.4	28.6	338.0	56.1	36.7	461.1	51.3	78.3	6.3	696.8	...
B	—	—	—	179.0	—	—	262.9	441.9	—	—	853.7	295.4	170.1	—	72.6	...
P	—	17.9	7.3	6.8	30.7	54.6	120.0	237.3	140.5	55.0	—	8.7	—	—	303.0	...
R	94.5	44.8	7.3	187.4	1.8	775.6	131.5	1,242.9	1,615.9	458.4	—	67.3	—	1,480.2	1,992.2	...
S	504.1	618.7	16.2	22.0	13.2	413.6	354.4	1,942.2	1,757.8	1,192.0	—	—	—	—	—	824.2
$\sum_i X_{ij}$	3,560.2	7,289.5	177.1	734.6	51.7	2,062.5	897.4	14,773.0	17,839.0	1,833.8	1,314.8	648.8	248.4	1,844.6	3,860.9	824.2
V_i	12,192.6	1,676.7	153.9	953.9	86.8	810.0	4,818.8									
X_i	15,752.8	8,966.2	331.0	1,688.5	138.5	2,872.5	5,716.2									

TABLE VII-B

POSSIBLE STATE OF AFFAIRS IN 1964/65 IN WEST PAKISTAN

	A	F	M	B	P	R	S	$\sum_j X_{ij}$	Private consumption	Public consumption	Fixed investment	Working capital increase	Replacement	Exports	Imports	Trade & transport into imports
A	1,652.5	5,277.0	117.7	78.2	—	1,092.0	—	8,217.4	5,766.8	—	—	129.1	—	329.1	1,327.0	...
F	314.8	72.7	—	—	—	20.2	—	407.7	7,439.3	163.0	—	117.3	—	193.4	239.5	
M	183.6	—	278.7	225.3	66.0	74.1	86.7	914.4	208.0	130.4	1,009.8	96.6	170.6	83.2	1,671.3	...
B	—	—	—	572.6	—	—	95.4	668.0	—	—	1,613.4	458.9	407.9	0.2	19.7	...
P	—	56.6	25.4	56.3	269.0	134.8	581.0	1,123.1	410.4	163.0	—	67.4	—	35.5	149.4	...
R	209.8	436.4	59.3	638.3	33.0	2,157.0	208.1	3,741.9	3,014.4	978.1	—	134.5	—	1,365.9	2,494.1	...
S	590.2	727.3	73.5	168.9	306.9	1,691.9	381.5	3,940.2	2,060.4	1,825.7	—	—	—	—	—	844.8
$\sum_i X_{ij}$	2,950.9	6,570.0	554.6	1,739.6	674.9	5,170.0	1,352.7	19,012.7	18,899.3	3,260.2	2,623.2	1,003.8	578.5	2,007.3	5,901.0	844.8
V_i	10,164.5	1,511.2	387.1	1,389.1	975.1	1,570.7	7,318.4									
X_i	13,115.4	8,081.2	941.7	3,128.7	1,650.0	6,740.7	8,671.1									

Note : **Imports and exports include interregional flows.**

TABLE VIII

COMPONENTS OF FINAL DEMAND

(million rupees)

Sector	West Pakistan			East Pakistan		
	dG	dE	dD	dG	dE	dD
A	—	108.4	—	—	85.9	—
F	69.0	260.9	—	38.8	102.0	—
M	55.2	—	185.2	15.5	—	40.2
B	—	—	9.3	—	—	28.6
P	69.0	1.2	—	23.3	—	—
R	358.7	568.5	—	194.1	525.3	—
S	827.9	—	—	504.5	—	—

TABLE IX

MINIMUM INCREASE IN CONSUMPTION OVER THE THIRD-PLAN PERIOD

(million rupees)

Sector	East Pakistan	West Pakistan
A	2,197.6	1,918.9
F	2,050.3	1,118.1
M	30.7	68.4
P	61.4	136.7
R	1,000.5	971.7
S	798.0	668.9

TABLE X

INCREMENTAL CAPITAL-VALUE-ADDED RATIOS

Sector	East Pakistan	West Pakistan
A	.931	1.925
F	2.235	2.182
M	5.996	5.599
B	1.970	3.164
P	10.181	7.648
R	3.468	3.532
S	3.854	2.852

TABLE XI

SOLUTION OF THE BASIC OPTIMIZING MODEL (BOM)

	Sector	East Pakistan	West Pakistan
OUTPUT	A	5,828.5	4,436.9
	F	2,231.4	1,503.6
	M	187.6	640.9
	B	3,892.4	2,770.3
	P	64.5	282.2
	R	672.2	4,243.9
	S	3,067.7	3,772.8
CONSUMPTION	A	2,197.6	1,991.7
	F	2,050.3	1,118.1
	M	30.7	68.4
	P	61.4	136.7
	R	1,000.5	971.7
	S	1,379.0	892.2
IMPORT	A	80.4	—
	M	389.5	622.7
	P	136.9	310.3
	R	1,997.3	203.6
Aggregate capital-income ratio		2.25	3.15
Incremental saving rate		24.5%	30.3%
Regional product (increase)		9,923.5	9,411.1
Gross investment		22,349.4	29,650.6
Increase in the use of foreign exchange		1,501.0	791.0

Note : All values are in million rupees at 1964/65 prices.

TABLE XII

SOLUTION OF THE W-MAXIMIZING MODEL

		East Pakistan	West Pakistan
OUTPUT	A	5,828.5	4,232.5
	F	2,232.7	1,497.1
	M	451.2	397.5
	B	4,484.3	1,876.9
	P	78.2	251.2
	R	1,853.9	3,984.8
	S	3,297.7	3,392.5
CONSUMPTION	A	2,197.6	1,918.9
	F	2,050.3	1,118.1
	M	30.7	68.4
	P	61.4	136.7
	R	1,000.5	971.7
	S	1,332.9	668.9
IMPORT	A	572.2	—
	M	596.2	441.6
	P	167.5	277.0
	R	1,039.1	438.7
Aggregate capital-income ratio		2.36	3.14
Incremental saving rate		31.8%	25.1%
Increase in GRP		10,915.9	8,361.6
Gross investment		25,728.4	26,271.6
Increase in the use of foreign exchange		1,545.1	746.9

Note : All values are in million rupees at 1964/65 prices.

TABLE XIII

SOLUTION OF THE GDP-MAXIMIZING MODEL

		East	West
OUTPUT	A	5,828.5	4,436.9
	F	2,231.4	1,505.4
	M	89.7	834.3
	B	2,655.9	3,835.7
	P	55.9	325.4
	R	644.9	4,818.2
	S	2,352.6	4,371.1
CONSUMPTION	A	2,356.2	1,918.9
	F	2,050.3	1,118.1
	M	30.7	68.4
	P	61.4	136.7
	R	1,000.5	971.7
	S	798.0	1,198.2
IMPORT	A	—	82.1
	M	185.6	941.4
	P	118.4	357.3
	R	1,892.1	95.4
Aggregate capital-income ratio		2.14	3.17
Increase in regional product		8,563.8	10,628.2
Gross investment		18,299.0	33,701.0
Increase in the use of foreign exchange		1,208.0	1,084.0

Note : All values are in million rupees at 1964/65 prices. Note that GDP for this exercise is actually .007 less than in the BOM. This slight discrepancy must be due to rounding error because this exercise can not give a smaller maximand than the BOM. We have made every effort to assure us that this is due to no other reason than rounding error made by the computer.

TABLE XIV

SOLUTION OF THE THIRD PLAN ALLOCATION MODEL

		East Pakistan	West Pakistan
OUTPUT	A	5,828.5	4,144.3
	F	1,362.2	1,477.5
	M	780.3	124.3
	B	4,436.9	1,671.8
	P	87.6	241.3
	R	3,038.1	3,858.9
	S	3,161.2	3,362.5
CONSUMPTION	A	2,197.6	1,918.9
	F	2,050.3	1,118.1
	M	30.7	68.4
	P	61.4	136.7
	R	1,000.5	971.7
	S	929.3	668.9
IMPORT	A	323.0	—
	F	827.4	—
	M	640.4	465.4
	P	190.1	266.5
	R	—	752.1
Aggregate capital-income ratio		2.43	3.11
Incremental saving rate		36.5%	22.0%
Increase in regional product		11,103.7	8,028.6
Gross investment		27,000.0	25,000.0
Increase in foreign exchange used		1,364.7	927.3

Note : All values are in million rupees at 1964/65 prices.

TABLE XV

INVESTMENT IN NON-AGRICULTURAL SECTORS : THIRD PLAN AND THE OPTIMIZING EXERCISES

Sector	Gross TFYP Investment		Net Investment in the			
			BOM		TFYP-allocation model	
	(million rupees)	(% of total)	*(million rupees)*	(% of total)	*(million rupees)*	(% of total)
M	1,208	2.3	1,998	4.2	2,461	5.2
P	4,533	8.7	1,660	3.5	1,613	3.4
R + F	10,993	21.1	5,696	12.1	7,319	15.5
S + B	22,916	44.1	27,273	57.9	25,651	54.5

TABLE XVI

THE 14 BALANCE EQUATIONS CAN BE WRITTEN, BY COLLECTING THE COEFFICIENTS, IN THE FORM AX + F = X.

WE GIVE BELOW THE ELEMENTS OF THE MATRIX A. ROWS AND COLUMNS ARE IN FOLLOWING ORDER: A, F, M, B, P, R, S, IN EAST FOLLOWED BY A, F, M, B, P, R, S, IN WEST

.221	.744	.271	.164	—	.272	—	.003	.015	.003	.001	—	.004	—
.005	.068	—	—	—	.001	—	—	—	—	—	—	—	—
.018	.033	.831	.059	.818	.116	.143	—	—	.006	.001	.005	.001	.001
.172	.015	.074	.431	.917	.071	.960	—	—	—	—	—	—	—
—	.002	.027	.004	.419	.020	.021	—	—	—	—	—	—	—
.019	.004	.022	.096	.015	.315	.019	.002	.003	.004	.011	.001	.022	.001
.034	.073	.061	.020	.054	.164	.065	—	—	—	—	—	—	—
.003	.009	.003	.002	—	.003	—	.146	.664	.129	.026	—	.170	—
—	.001	—	—	—	—	—	.026	.057	—	—	—	.003	—
.004	.007	.182	.013	.179	.026	.031	.059	.040	.967	.114	.884	.117	.197
—	—	—	—	—	—	—	.407	.019	.064	.591	.587	.044	.623
—	—	—	—	.008	—	—	—	.007	.033	.019	.230	.021	.069
.005	.001	.005	.023	.004	.076	.004	.046	.053	.073	.207	.023	.409	.024
—	—	—	—	—	—	—	.046	.093	.079	.057	.186	.257	.044

Note: — means that the coefficient is less than .0005