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Estimates of Farm Output Supply and Input Demand Elasticities: The Translog Profit Function Approach

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The importance of estimating valid elasticities of farm output supply and input demand can hardly be overemphasised. Reliable estimates of these elasticities are sine qua non for predicting accurately the farmer responsiveness to changes in input-output prices and government taxes and thereby for formulating successful agricultural incentive programmes consistent with national requirements of food, development and exports. In fact, robust estimates of the coefficients of such elasticities can serve as a solid basis in determining effective policy relevant interventions for promoting production, equity, efficiency, and finally egalitarian income distribution in the farm sector of the economy.

Farmer input factor demand and output supply elasticities have earlier been derived with direct or indirect application of the Cobb-Douglas production function to farm survey data [Lau and Yotopoulos (1971); Yotopoulos (1972); Yotopoulos, Lau and Lin (1976); Junankar (1980) and Sindhu and Baanannte (1981)]. The Cobb-Douglas production function is based on highly restricted assumptions such as the unitary elasticity of substitution, constant returns to scale and "a priori" imposition of separability restrictions. Therefore, it yields invalid elasticities which fail to explain the genuine relationships between inputs and outputs [Diewert (1971); Christensen, Jorgenson and Lau (1971)]. Further, such elasticities have also been estimated with the constant elasticity of substitution production function (CES), variable elasticity of substitution production function (VES), and the nested-CES production function applied to time-series data [Battese and Malik (1988); Berndt and Christensen

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(1973); Narasimham and Fabryey (1974); Trosper (1978); Mousa and Jones (1991)]. These production functions, though regarded superior to the Cobb-Douglas production function, are also based on rigid restrictions. For example, CES production function operates with only two independent variables and yields a constant, though not unitary, value of the elasticity of substitution for all levels of inputs and outputs whereas its "nested" form involves arbitrary aggregation of two of the independent variables into one variable to be combined with the third as the second independent variable. Further, since both the CES and VES are non-linear production functions, their estimation involves excessive iteration. Further still, the elasticity of substitution is variant in VES but invariant in CES with respect to changes in capital-labour ratio. As such, all these production functions being characterised by specific shortcomings are incapable of explaining exact relationships among variables.

Some recent studies have applied flexible functional forms in analysing the farmer production structure, input demand and output supply [Sindhu and Baanannte (1981); Mckay, Laurance and Vlastuin (1983)]. So far, in Pakistan only one study [Ali and Parikh (1992)] has examined the relationships among different inputs by applying the translog cost function to a small specific farm survey data set. Although this is an excellent study, its farm coverage is extremely limited. Further, its use of cost function rather than profit function has precluded the provision of information on equally important farm output supply relationships.

As such, this study has, for the purpose of ascertaining the relationships among different inputs and outputs, estimated their demand and supply elasticities with a flexible profit function. Its superiority over other functional forms lies in its certain important properties. Its estimation imposes no restrictions, it integrates the input demand functions with the output supply function and it uses the input prices rather than input quantities for which reliable information is obtainable from both the farm and non-farm sources and they do not involve the problem of aggregation which is commonly associated with input quantities.

Generalisation of Normalised Restricted Translog Profit Function

Assuming a farmer maximises profit subject to a given state of technology and a mix of fixed inputs, and marginal conditions hold, the normalised restricted translog profit function for one output, as formulated by Christensen, Jorgenson and Lau (1971), is depicted below:[1]

[1]Working with the profit function depicted by Equation (1) has several advantages over working with the production function [Lau and Yotopoulos (1971)]. The profit function, the supply function, and the derived demand functions are functions only of the normalised input prices and quantities of fixed inputs, which are argued normally to be determined independently of producer's behaviour. Since these variables are exogenous, estimating functions containing them avoids the problem of simultaneous equations bias that arises in single equation production function estimation, because the quantities of the independent variable factors, as indicated by the conditions of profit maximisation, are not truly exogenous.

$$\ln \pi^* = \alpha_0 + \sum_{i=1}^{n} \alpha_i lnP_i^* + 1/2 \sum_{i=1}^{n} \sum_{h=1}^{n} \gamma_{ih} lnP_i^* lnP_h^*$$
$$+ \sum_{i=1}^{n} \sum_{k=1}^{m} \delta_{ik} lnP_i^* lnZ_k + \sum_{k=1}^{m} \beta_k lnZ_k$$
$$+ 1/2 \sum_{k=1}^{m} \sum_{j=1}^{m} \Phi lnZ_k lnZ_j \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where, π^* is the normalised restricted profit derived as total revenue minus total variable costs of variable inputs divided by the output price. P_i is the price of variable input, X_i, again normalised by output price, Z_k is the Kth fixed input, whereas i=h=1,2,3……, n; k = j = 1,2,3……, m and α_o,α_i,γ_{ih}, δ_{ik} β_k and ϕ are parameters.

There is a duality of the profit function with a corresponding production possibility set. To ensure the existence of the duality between them, it is sufficient for the profit function (π (P, Z)) to be (i) a non-negative real value function defined for all $P > 0$ and any Z; (ii) homogenous of degree one in P, (iii) convex and continuous in P for every fixed Z, (iv) homogenous of degree one in Z, (v) concave and continuous in Z for every fixed P, (vi) non-decreasing in Z for every fixed P and non-decreasing (non-increasing) in P_i if i is an output (variable input) for every Z.

If the above conditions hold and producers maximise profits, then the profit function contains sufficient information to completely describe the production technology and hence the production possibility set [Diewert (1974)]. These properties of the profit function correspond generally to the more familiar properties of the dual production possibility set. For example, the condition that $\pi(P, Z)$ is concave in Z for every P corresponds to the assumption of non-increasing marginal rate of transformation between pairs of inputs. Similarly, if the production technology exhibits constant returns to scale, the profit function will be homogenous of degree one in fixed inputs as mentioned above in the condition (iv) of the duality of the profit function.[2]

To explain the application of the translog profit function further, let us define $S_i \equiv \frac{-P_i\ X_i}{\pi^*}$ as negative of the ratio of variable expenditures for the ith input relative to the restricted profit. Similarly, let $S_v \equiv \frac{V}{\pi^*}$ to be the ratio of output supply (v) to the normalised restricted profit. It may be seen that according to Diewert (1974), and Christensen, Jorgenson and Lau (1971), differentiating the translog profit function (1) with respect to P_i^* and to the price of output yields a system of variable

[2]It may, however, be noted here that although the conditions of the duality of the profit function mentioned above are sufficient, they are not necessary conditions. For example, according to Lau (1969) duality does not require the profit function to be homogenous of degree one in fixed input, which also means that the constant returns to scale of the production technology are not a necessary condition for it.

input/profit ratio and an output/profit ratio. However, since S_i and S_v sum to unity, one of the equations can be ignored, and ignoring the output share equation and using Hotelling Lemma, the translog profit Equation (1) can serve to obtain the following share equation:

$$S_i = -\frac{P_i^* X_i}{\pi^*} = \frac{\partial \ln \pi^*}{\partial \ln P_i^*}$$

$$S_i = \alpha_i + \sum_{h=i}^{n} \gamma_{ih} \ln P_h^* + \sum_{k=1}^{m} \delta_{ik} \ln Z_k \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

It may be noted that in this way we are able to simultaneously estimate both the profit and variable inputs share equations as function of the normalised input prices and quantities of fixed factors as exogenous variables. It may similarly be noted further that once the parameter estimates of Equations (1) and (2) are obtained, the elasticities of variable input demands and output supply with respect to all the exogenous variables, evaluated at averages of the relevant S_i and, at given levels of variable input prices, are then the linear transformations of the profit function parameter estimates. Following Sindhu and Baanannte (1981), variable input demands and output supply elasticities with respect to input prices, output price, and fixed inputs can be obtained from the model. Own-price elasticities for variable input demand function are expressed as follows:

$$\eta_{ii} = -\ S_i^* - 1 - \frac{\gamma_{ii}}{S_i^*} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

Similarly, the cross-price elasticity of demand (η_{ih}) for input $i \neq h$ with respect to the price of hth input can be obtained as explained below:

$$\eta_{ih} = -\ S_h^* - \frac{\gamma_{ih}}{S_i^*} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

Further, the elasticity of demand for input i (η_{iy}) with respect to output price, P_y, can also be obtained as shown below:

$$\eta_{iy} = \sum_{i=1}^{n} S_i^* + 1 + \sum_{h=1}^{n} \frac{\gamma_{ih}}{S_i^*} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

Finally, it is also possible to derive the price elasticity of demand, (η_{ik}), for input i with respect to the kth fixed factor, Z_k, as given below:

$$\eta_{ik} = \sum_{i=1}^{n} \delta_{ik} \ln P_i + \beta_K - \frac{\delta_{ik}}{S_i^*} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

Like the determination of the elasticities of the variable input demands with respect to the exogenous variables expressed as the linear transformation of the parameter estimates of profit function, output supply elasticities with respect to output price, prices of variable inputs and quantities of fixed factors, evaluated at given levels of exogenous variables, can also be expressed as linear functions of the restricted profit function parameters. The elasticity of supply (ε_{vi}) with respect to *i*th variable input price is, in turn, given by the following equation:

$$\varepsilon_{vi} = -S_i^* - \sum_{h=1}^{n} \gamma_{hi} /(1+\sum_{h=1}^{n} S_h^*) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

Further, the own-price elasticity of supply (ε_{vv}) is given similarly by the equation mentioned below:

$$\varepsilon_{vv} = \sum_{i=1}^{n} S_i^* + \sum_{i=1}^{n} \sum_{h=1}^{n} \gamma_{hi} /(1+\sum_{h=1}^{n} S_h^*) \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

Finally, the elasticity of output supply (ε_{vk}) with respect to the fixed inputs, Z_K, is given by the following equation:

$$\varepsilon_{vk} = \sum_{i=1}^{n} \delta_{ik} lnP_i + \beta_k - \sum_{i=1}^{n} \delta_{ik} /(1+\sum_{h=1}^{n} S_h^*) \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

Estimation Procedure

The preceding model constitutes the basis for empirical application of the translog profit function for which labour and fertiliser are assumed as variable inputs, and capital, livestock, land, and education are assumed as quasi fixed inputs. The price of labour used in this estimation is daily wage rate and the price of fertiliser is the money price per kilogram of fertiliser both normalised by the price of output. Further, the share equations for labour and fertiliser and obtained by differentiating the normalised restricted translog profit function represented by Equation (1). S_I represented by Equation (2) are estimated for whole of Punjab and its zones of cotton, rice and mixed cropping.

The above model represented by Equations (1) and (2) are estimated jointly. In estimating these equations, the error terms for each equation are assumed to satisfy all classical assumptions but correlated across equations. It may be noted that the cross equation restrictions cannot be imposed in equation by equation estimation by OLS or by 2SLS. Thus, the appropriate estimation procedure as applied here is to perform a constrained three-equation multivariate regression by minimising the trace of residual covariance matrix, that is, estimating a stacked regression by OLS or 2SLS. If the off-diagonal elements of the resudual covariance matrix are non-zero, greater efficiency in estimation may be obtained and again which is done in the analysis by taking the estimated covariances into account and using Zellner's Efficient Estimation procedure.

Suppose if one suspected that input prices were endogenous and that failure to account for this might lead to a simultaneous equations bias, 2SLS is suggested to obtain appropriate estimates. Additional gain in estimation efficiency may also be obtained by treating a set of equations estimated by 2SLS as "seemingly unrelated" and then applying the three stage least squares (3SLS) technique. This additional estimation has also been carried out. Iterating on 3SLS procedure leads to estimates, which are asymptotically invariant to the choice of equation to be dropped, because this procedure is asymptotically equivalent to the full information maximum likelihood estimation [Berndt and Christensen (1973)].

The same estimation can alternatively be carried out by applying the Maximum Likelihood Estimation Technique (MLE). Barten (1969) has applied MLE in investigating properties of a system with the stochastic specification of our translog equations. However, Kementa and Gilbert (1968) have shown that iterating Zellner Efficient Estimation (ZEF) converges the parameter estimates to the Maximum Likelihood Estimates. Since ZEF method is computationally equivalent to MLE, and is performed in the Time-series processor (TSP) package, this study uses the Iterative Zellner Efficient (IZEF) method.

To confirm the reliability of the input demand and output supply elasticities, this analysis is carried out in a way that the estimates are also verified by tests applied to two separate hypotheses. The first empirical test checks the validity of the symmetry and homogeneity restrictions across profit and S_i equations. This is a joint hypothesis and the validity of the constraints implies that, the average sample farmer, maximise profit with respect to the normalised variable input prices. The curvature requirements are also tested by calculating the Eigen values from the Hessian matrix. The second test verifies the relevance of the Cobb-Douglas (C-D function) vis-à-vis the translog profit function for the present analysis. Theoretically, it is hypothesised that for a profit function to be Cobb-Douglas in nature, coefficients of all the second order terms should be zero. The rejection of this hypothesis on the basis of Log-likelihood ratio test will declare the translog function as superior to the C-D function for this analysis.

The coefficients of the translog profit function and S_i functions are used to derive the elasticities of output supply and input demand for variable inputs used in the analysis. More specifically, these elasticities are functions of variable input prices, variable input ratios, level of fixed inputs and the translog profit function parameter estimates. All estimates of these elasticities are evaluated at the mean of the data set.

DATA SOURCES

The study is based on a sample farm survey of irrigated areas of Punjab. In all, 484 farmers were personally interviewed from 20 villages scattered in three well-defined cropping zones: cotton zone, rice zone and mixed cropping zone.[3] The sample farmers were distributed by size and tenure roughly in proportion to their population in the province as shown in the Table 1 below:

[3]The rice zone spans the districts of Sialkot, Gujranwala and Sheikhupura. The cotton zone, on the other hand, comprises Multan, Khanewal, Vehari, Rahim Yar Khan, Jhang, Muzaffar Garh and D. G. Khan. Finally, the mixed cropping zone includes Faisalabad, Sahiwal, Sargodha, Mianwali, and Khushab.

Table 1

Distribution of Sample Farmers by Size and Tenure

Farm Size	Respondents			
	Owner Operative	Owner cum Tenants	Tenants	Total
Under 12.5 Acres	161	55	76	292
12.5–25 Acres	81	26	38	145
25–50 Acres	24	10	13	47
Total	266	41	127	484

Source: Farm Survey Data.

Although the span of one cropping zone varies widely from the other, same number of framers were interviewed from each zone primarily because framing practices in them appear to be at nearly the same stage of advancement. As such, the distribution of the sample framers by size of their holding and tenurial status in each cropping zone was as depicted below:

Table 2

Distribution of Survey Farmers by Cropping Zones

Farm Size	Farm Tenure			
	Owner Operative	Owner cum Tenants	Tenants	Total
Under 12.5 Acres	55	17	25	97
12.5–25 Acres	27	9	12	48
25–50 Acres	9	3	4	16
Total	91	29	41	161

Source: Farm Survey Data.

The survey data analysed was collected by trained enumerators from October 1991 to September 1992 through the pre-tested questionnaires. The analysis has been performed by each and all zones together. For econometric estimation, labour and fertiliser are used as variable inputs and capital (farm equipment, machinery, building, etc.), livestock, land and household education as fixed inputs. These variables are defined as below:

1. **Restricted Profit:** As discussed before, restricted profit, π^*, used in the analysis is defined as output revenue minus variable costs. The restricted profit normalised by output price is expressed as a function of the normalised variable input prices and the fixed input quantities.
2. **Labour:** Labour input used in the analysis comprised the family labour and the casually hired labour. It is transformed into monetary expenditure by adding the imputed cost of family labour at the permanently hired

labour wage rate in the area concerned to the expenditure on the casual and/or the regularly hired labour. Child and female labour is converted into man-equivalents by treating two children or two women equal to one man. The weighted average of the daily money wage rate was calculated by dividing the total labour expenditure by the labour days employed per farm. P_L^* appearing in the empirical functions is the weighted average of money wage rate of labour per day normalised by the output price.

3. **Fertiliser:** Fertiliser input per farm is measured as money price of fertiliser per kilogram normalised by the output price. It is a weighted price per kilogram of fertiliser worked for all kinds of fertilisers used by the sample farmers. More specifically, it is obtained by dividing the total fertiliser expenditure per farm by the total kilograms of fertiliser used. It appears as P_F^* in different functions.
4. **Capital:** The variable of capital is used in this analysis as rupees of annualised flow cost of the total investment expenditure in equipment, machinery, implements, buildings, etc., per farm and represented by Z_1 in equations. The annuities and depreciation are calculated at the rate of 10 percent and 5 percent respectively.
5. **Livestock:** The livestock input is measured as annualised flow of animal capital at 10 percent rate of interest and represented in numerical function by Z_2.
6. **Land:** Land input is denoted by Z_3 in regression equations. It is measured as the cultivated land acres operated by sample households.
7. **Education:** It is measured as the average number of years of schooling per family member above 13 years of age. It is indicated by Z_4 in functions.

Output Supply and Input Demand Elasticities

As mentioned before, the farmer output supply and input demand elasticities are derived with the translog profit function and S_i functions for each and all cropping zones together for knowing the deviations from the aggregate sample. The required input demand and output supply elasticities are determined by combining the estimates of the translog profit function and S_i functions with the variable input ratios, variable input prices and quantities of fixed inputs. The parameter estimates of the translog profit function for whole of Punjab are depicted in Table 3 and the elasticities derived from them in Table 4.

While the estimates of output and input are important in many ways, their real usefulness depends on their statistical significance and theoretical consistency. Viewed from this aspect, Table 4 represents the coefficients of labour and fertiliser which are statistically significant and carry valid signs and sizes. Their reliability has also been established by their Cobb-Douglas functions estimates. The same table shows that the impact of a given change in any of the exogenous variable and fixed

Table 3

Restricted Parameter Estimates of the Translog Profit Function for Punjab Agriculture

	Intercept	Price of Labour LnP_L	Price of Fertiliser LnP_F	Capital $Ln Z_1$	Livestock $Ln Z_2$	Land $Ln Z_3$	Education $Ln Z_4$
Profit Function	5.40	–3.00**	–0.45*	0.13	0.93*	0.15*	0.90
	(7.41)	(1.61)	(0.06)	(0.15)	(0.19)	(0.06)	(0.84)
Share Equation of Labour	–2.97**	–0.28	0.11**	0.05*	0.09	0.02	0.24**
	(1.61)	(0.21)	(0.06)	(0.01)	(0.15)	(0.17)	(0.13)
Share Equation of Fertiliser	–0.45*	–0.40*	0.11	–0.22*	0.30*	0.01	0.17
	(0.06)	(0.12)	(0.26)	(0.06)	(0.13)	(0.14)	(0.13)
	$(Ln P_L)^2/2$	$(Ln P_F)^2/2$	LnP_LLnP_F	LnP_LLnZ_1	LnP_LLnZ_2	LnP_LLnZ_3	LnP_LLnZ_4
	–0.28	–0.41*	0.11*	0.05	0.09	0.02	0.24**
	(0.21)	(0.12)	(0.06)	(0.06)	(0.15)	(0.17)	(0.13)
	LnP_FLnZ_1	LnP_FLnZ_2	LnP_FLnZ_3	LnP_FLnZ_4	$(Ln Z_1)^2/2$	$(Ln Z_2)^2/2$	$(Ln Z_3)^2/2$
	–0.22*	0.30*	0.10	0.17	–0.01	0.04	0.90**
	(0.06)	(0.13)	(0.14)	(0.13)	(0.01)	(0.03)	(0.05)
	$(Ln Z_4)^2/2$	LnZ_1LnZ_2	LnZ_1LnZ_3	LnZ_2LnZ_3	LnZ_2LnZ_4	LnZ_3LnZ_4	LnZ_1LnZ_4
	–0.02*	0.02	0.04	–0.001	–0.04	–0.04	–0.01
	(0.07)	(0.03)	(0.04)	(0.07)	(0.08)	(0.06)	(0.03)

*Significant at 0.05 level. **Significant at 0.01 level.

Note: Asymptotic standard errors are in parenthesis.
Sample range 1...484.
Pseudo R^2=0.38.

Table 4

Derived Elasticity Estimate of Output and Demand for Variable Inputs for Punjab Agriculture

	Price of Output	Price of Labour	Price of Fertiliser	Capital	Livestock	Land	Education
	Derived from Parameter Estimates Translog Profit Function						
Output	0.72	–0.71	–0.8	0.16	0.09	0.79	0.09
Labour	1.29	–1.03	–0.64	0.69	0.32	0.01	0.35
Fertiliser	1.2	–0.72	–1.46	0.64	0.11	0.66	0.34
	Derived from Parameter Estimates Cobb-Douglas Profit Function						
Output	1.12	0.48	0.64	0.13	0.15	0.57	0.15
Labour	2.12	–0.48	–1.64	0.13	0.15	0.57	0.15
Fertiliser	2.12	–1.48	–0.64	0.13	0.15	0.57	0.15

Source: Derived from Survey Data.

inputs across input demand functions of labour and fertiliser in C-D case is symmetric because of its yielding unitary elasticity of substitution among all input paris while it varies in case of a translog function. In fact, the non-symmetric effects of all the prices included in the translog profit function are consistent with their theoretical behaviour hypotheses. The superiority of the translog profit function over the C-D production function may more specifically be reflected from the fact that a 10 percent increase in output price increases the demand for labour by 13 percent and that of fertiliser by 12 percent compared to a symmetric increase of 21 percent in their demand shown by C-D function.

The elasticities of the labour demand with respect to wage rate and fertiliser price are –1.677 and –0.6, respectively. These negative elasticities indicate that fertiliser and labour are complementary inputs. The complementarity in the relationship of labour and fertiliser has important implications. Their combined application increases farm production synergistically. Further, additional labour out of the unemployed workforce could be employed in agriculture by increasing its level of fertiliser application. Elasticities of labour with respect to capital, livestock, land and education are positive. Although they are inelastic, they do indicate a fair degree of farm sector responsiveness to labour absorption and a potential in need of realisation with effective policy measures such as increased public investment in irrigation, roads and other infrastructure. Similarly, increased fertiliser application also deserves consideration. Its own price elasticity is greater than unity. Its price elasticities with respect to output price is greater and those with respect to capital, livestock, land and education are less than unity but all are positive. As such, all these carry good policy relevance. It may, for example, be seen that a 1 percent exogenous increase in education of a farm household increases its demand for fertiliser by 0.09 percent. Exogenous increases in fixed factors affect farm production positively. Alternatively expressed, increases in fixed factors shift the marginal productivity curves of labour and other factors of production up.

The Log-likelihood ratio test of profit maximisation, performed in the form of a joint hypothesis of the validity of imposing restrictions on simultaneous estimation of Equations (1) and (2) of the model yielded the value of 37.5 which compared with the tabulated value of χ^2 does not reject the null hypothesis of the validity of the restrictions at 0.05 level of significance meaning that the sample farms maximise profits from farm production. The Log-likelihood ratio test carried out to check the validity of the C-D production function rejects it as the relevant approach for this analysis on the basis of the calculated Log-likelihood ratio value of 45.7 being in excess of its table value of χ^2. Curvature conditions were also satisfied for all the models.

Output Supply and Input Demand Elasticities for Rice Belt, Punjab

The restricted translog function parameter estimates and the elasticities derived from them for the farmers of this belt are presented in Table 5 and 6, respectively.

Table 5

Restricted Parameter Estimates of the Translog Profit Function, Rice Belt, Punjab

	Intercept	Price of Labour LnP_L	Price of Fertiliser LnP_F	Capital $Ln Z_1$	Livestock $Ln Z_2$	Land $Ln Z_3$	Education $Ln Z_4$
Profit Function	–18.57	–2.70*	–4.36**	0.85*	0.07**	1.26*	0.16*
	(14.68)	(1.13)	(2.45)	(0.21)	(0.04)	(0.23)	(0.01)
Share Equation of Labour	–2.70	–0.21	–0.27*	0.09*	0.07	0.22*	0.13*
	(2.82)	(0.39)	(0.09)	(0.01)	(0.21)	(0.06)	(0.03)
Share Equation of Fertiliser	–4.36**	0.22**	–0.27	–0.04	0.48*	0.19	–0.12**
	(2.45)	(0.11)	(0.09)	(0.13)	(0.19)	(0.26)	(0.06)
	$(Ln P_L)^2/2$	$(Ln P_F)^2/2$	LnP_LLnP_F	LnP_LLnZ_1	LnP_LLnZ_2	LnP_LLnZ_3	LnP_LLnZ_4
	–0.21	–0.22**	–0.27	–0.09*	–0.07	–0.22*	0.13
	(0.39)	(0.11)	(0.53)	(0.01)	(0.21)	(0.06)	(0.31)
	LnP_FLnZ_1	LnP_FLnZ_2	LnP_FLnZ_3	LnP_FLnZ_4	$(Ln Z_1)^2/2$	$(Ln Z_2)^2/2$	$(Ln Z_3)^2/2$
	–0.04	–0.48*	0.19	–0.12**	0.03	0.01	0.09
	(0.13)	(0.19)	(0.26)	(0.06)	(0.02)	(0.08)	(0.09)
	$(Ln Z_4)^2/2$	LnZ_1LnZ_2	LnZ_1LnZ_3	LnZ_2LnZ_3	LnZ_2LnZ_4	LnZ_3LnZ_4	LnZ_1LnZ_4
	–0.20**	–0.11**	0.01	0.14	0.01	0.13	–0.04
	(0.11)	(0.07)	(0.07)	(0.15)	(0.16)	(0.11)	(0.07)

*Significant at 0.05 level. **Significant at 0.01 level.

Note: Asymptotic standard errors are in parenthesis.
Sample range 1…168.
Pseudo R^2=0.35.

Table 6

Derived Elasticity Estimate for Rice Supply and Demand for Variable Inputs of Rice Belt, Punjab

	Price of Output	Price of Labour	Price of Fertiliser	Capital	Livestock	Land	Education
	Derived from Parameter Estimates Translog Profit Function						
Output	1.45	–0.71	–0.63	0.3	0.72	0.69	0.04
Labour	1.12	–1.9	–0.98	0.9	0.1	0.08	0.3
Fertiliser	1.66	–1.11	–1.94	–0.07	0.62	0.85	0.71
	Derived from Parameter Estimates Cobb-Douglas Profit Function						
Output	0.66	–0.17	–0.49	0.24	0.18	0.56	0.02
Labour	1.66	–1.17	–0.49	0.24	0.18	0.56	0.02
Fertiliser	2.66	–0.17	–0.49	0.24	0.18	0.56	0.02

Source: Derived from Survey Data.

The relevance of the translog profit function over the C-D function in analysing rice farmer survey data has been established by both the test of profit-maximisation behaviour of the sample farmers based on constraints imposed on the parameters of profit function and S_i equations, and the rejection of null hypothesis about the second order coefficients of the C-D profit function in accordance with the Log-likelihood ratio test. As to the farmer input factor responsiveness to changes in prices, rice output with own-price elasticity of 1.5 has responded elastically to changes in its prices. Similarly, the farmers also demanded labour and fertiliser elastically owing to their own-price elasticities greater than unities. It is not only that their elasticities were numerically greater than unity but were also compatible with theoretical expectations. It is also important to note that the sizes of these coefficients are significantly bigger than those obtained for them from the aggregate sample. Further, the output supply elasticity with respect to fertiliser price is –0.6, which means that, if fertiliser prices per kilogram increase (decrease by 10 percent, rice output supply is likely to increase (decrease) by 6.3 percent. This combined with the own-price elasticity of 1.5 calls for a policy mix of raising the price of output and lowering the price of fertiliser which have synergistic effect on rice output.

The cross-price elasticity of variable input of labour with respect to fertiliser is negative at –0.98. Thus, labour and fertiliser are complementary to each other in production which means that increase in fertiliser use generates on-farm employment opportunities. Similarly, elasticity of fertiliser with respect to land and education is 0.9 and 0.7, respectively purporting that exogenous expansions in cultivated area and farm household education will tend to increase the demand for both fertiliser and labour. Like the own-price elasticities, the cross-elasticities of fertiliser with respect to labour, land and education are numerically bigger than those yielded by the overall sample.

The elascitiy of demand for labour with respect to the price of output is elastic. Specifically, a 10 percent increase in the output price is expected to increase the demand for labour by 15 percent. The elasticities of labour with respect to capital, livestock, land and education are positive. It means that the farm labour demand will increase but inelastically on exogenous increases in the fixed input factors and this seems to be more so in the rice belt of the Punjab province.

Output Supply and Input Demand Elasticities for Cotton Belt, Punjab

Cotton farmers profit function estimates and the estimated elasticities are given in Table 7 and 8, respectively. The validity of these estimates has been verified by the relevance of the translog profit function established by the parametric restrictions imposed as in other cases before. Almost all the own-price elasticities have the expected signs and sizes, and most of them are statistically significant at 5 percent level of significance. Specifically, the own-price elasticity of cotton output

Table 7

Restricted Parameter Estimates of the Translog Profit Function for Cotton Belt, Punjab

	Intercept	Price of Labour LnP$_L$	Price of Fertiliser LnP$_F$	Capital Ln Z$_1$	Livestock Ln Z$_2$	Land Ln Z$_3$	Education Ln Z$_4$
Profit Function	8.16	–1.56*	–0.65**	0.45*	0.76*	0.79**	0.70**
	(18.74)	(0.50)	(0.14)	(0.05)	(0.01)	(0.47)	(0.40)
Share Equation of Labour	–1.56*	–0.46**	–0.56	–0.12	–0.23*	0.43*	0.01
	(0.50)	(0.28)	(0.57)	(0.09)	(0.02)	(0.08)	(0.19)
Share Equation of Fertiliser	–0.65*	0.10	–0.55	–0.16**	0.10**	0.01	–0.11*
	(0.02)	(0.56)	(0.56)	(0.10)	(0.06)	(0.03)	(0.03)
	$(\ln P_L)^2/2$	$(\ln P_F)^2/2$	LnP$_L$LnP$_F$	LnP$_L$LnZ$_1$	LnP$_L$LnZ$_2$	LnP$_L$LnZ$_3$	LnP$_L$LnZ$_4$
	–0.46**	–0.08	–0.56**	–0.12**	–0.23**	0.42	0.01
	(0.28)	(0.28)	(0.04)	(0.06)	(0.12)	(0.28)	(0.19)
	LnP$_F$LnZ$_1$	LnP$_F$LnZ$_2$	LnP$_F$LnZ$_3$	LnP$_F$LnZ$_4$	$(\ln Z_1)^2/2$	$(\ln Z_2)^2/2$	$(\ln Z_3)^2/2$
	–0.16	0.10	0.01	–0.11	–0.02	0.08	–0.08
	(0.20)	(0.36)	(0.46)	(0.28)	(0.02)	(0.08)	(0.12)
	$(\ln Z_4)^2/2$	LnZ$_1$LnZ$_2$	LnZ$_1$LnZ$_3$	LnZ$_2$LnZ$_3$	LnZ$_2$LnZ$_4$	LnZ$_3$LnZ$_4$	LnZ$_1$LnZ$_4$
	–0.08	–0.02	0.09	–0.05	0.16	0.01	–0.01
	(0.11	(0.07)	(0.07)	(0.14)	(0.15)	(0.14)	(0.04)

*Significant at 0.05 level. **Significant at 0.01 level.

Note: Asymptotic standard errors are in parenthesis.
Sample range 169…336.
Pseudo R^2=0.45.

Table 8

Derived Elasticity Estimate for Cotton Supply and Demand for Variable Inputs of Cotton Belt, Punjab

	Price of Output	Price of Labour	Price of Fertiliser	Capital	Livestock	Land	Education
Derived from Parameter Estimates Translog Profit Function							
Output	1.56	–0.47	–0.61	0.29	0.36	0.66	0.08
Labour	1.53	–0.56	–0.83	0.30	0.20	0.17	0.21
Fertiliser	1.38	–0.93	–0.27	0.43	0.21	0.85	0.26
Derived from Parameter Estimates Cobb-Douglas Profit Function							
Output	0.14	–0.05	–0.09	0.11	0.01	0.75	0.15
Labour	1.14	–1.05	–0.09	0.11	0.01	0.75	0.15
Fertiliser	1.14	–0.05	–1.09	0.11	0.01	0.75	0.15

Source: Derived from Survey Data.

and the elasticities of fertiliser and labour are greater than unity, implying active price-responsiveness of cotton growers. By comparison, the numerical sizes of these elasticities are bigger than those for the aggregate sample. However, fertiliser own-price elasticity for cotton farmers is significantly less than that of the rice farmers. The cross-price elasticities of variable inputs of labour and fertiliser are negative confirming once again the complementarity relationships between them. It is ascertainable from the figures that 1 percent decrease in fertiliser price leads to 0.3 percent increase in its use, 0.6 percent increase in cotton supply and 0.8 percent increase in the labour demand. Expansion in education of the farm family is important. It increases the demand for labour and the use of fertiliser, which is turn contribute positively to the cotton output supply. Similarly, an exogenous increase in capital—farm machinery—increases the demand for labour and labour.

The elasticities of output with respect to wage rate and fertiliser price are negative but those with respect to fixed factors are positive but all less than unity. Consequently, increased availability of fixed factors induces expansion in use of variable factors and thereby in farm production. For example, a 1 percent expansion is land and capital will increase output by 0.7 percent and 0.3 percent, respectively, holding wage rate and price of fertiliser constant. The elasticity of labour with respect to wage rate is negative and less than unity in absolute terms but that of fertiliser with respect to output price is greater than one. The negative signs of the cross-price elasticities of labour and fertiliser indicate them as complementary factors of production.

Output Supply and Input Demand Elasticities for Mixed Cropping Belt of Punjab

Just as in earlier two cases, Log-likelihood ratio test of restrictions has proved the translog profit function as superior to the C-D, function for testing the profit maximising behaviour of the mixed cropping zone farmers. The results of the translog profit function and the estimates of the elasticities are presented in Table 9 and 10, respectively. A glance over these tables reveals that the own-price elasticity of output is less than unity. This seems due primarily to the dominance of wheat as a staple crop in this zone. However, the own-price elasticities of labour and fertiliser are of great importance. It may be realised that a 10 percent decrease in the price of fertiliser increases the output supply by 5.3 percent, the demand for labour by 3.1 percent and use of fertiliser by 13.2 percent. However, a similar increase in the price of output increases its supply by 6.3 percent, and the demand for labour and fertiliser by 12.5 percent and 14.1 percent, respectively. As mentioned before, elasticity estimates with respect to fixed factors are also important. Exogenous increases in the quantities of the fixed factors especially education and capital contribute generally positively to the supply of output by raising the demand for labour and fertiliser. The magnitude of the response in rice zone differs by some small margin in certain cases and by somewhat bigger margin in other zones but the direction has been the same and consistent with theoretical expectations.

Table 9

Restricted Parameter Estimates of the Translog Profit Function for Mixed Cropping Belt, Punjab

	Intercept	Price of Labour LnP_L	Price of Fertiliser LnP_F	Capital $Ln Z_1$	Livestock $Ln Z_2$	Land $Ln Z_3$	Education $Ln Z_4$
Profit Function	12.47	–7.83**	–1.34*	0.39*	0.14	0.08	0.09
	(11.82)	(3.94)	(0.40)	(0.14)	(1.43)	(1.12)	(0.17)
Share Equation of Labour	–7.63**	–0.12	0.26*	0.20**	0.45*	–0.44*	0.85*
	(3.94)	(0.55)	(0.01)	(0.11)	(0.03)	(0.06)	(0.35)
Share Equation of Fertiliser	–1.34*	–0.25*	0.26*	–0.39*	0.29*	0.29	–0.11
	(0.40)	(0.12)	(0.04)	(0.11)	(0.07)	(0.27)	(0.26)
	$(Ln P_L)^2/2$	$(Ln P_F)^2/2$	LnP_LLnP_F	LnP_LLnZ_1	LnP_LLnZ_2	LnP_LLnZ_3	LnP_LLnZ_4
	–0.12	0.25	–0.26	0.20**	0.45*	–0.44*	0.85*
	(0.55)	(0.18)	(0.50)	(0.11)	(0.03)	(0.06)	(0.35)
	LnP_FLnZ_1	LnP_FLnZ_2	LnP_FLnZ_3	LnP_FLnZ_4	$(Ln Z_1)^2/2$	$(Ln Z_2)^2/2$	$(Ln Z_3)^2/2$
	–0.39*	0.29**	0.29	–0.11	0.01	0.04	0.16**
	(0.11)	(0.17)	(0.27)	(0.27)	(0.02)	(0.06)	(0.08)
	$(Ln Z_4)^2/2$	LnZ_1LnZ_2	LnZ_1LnZ_3	LnZ_2LnZ_3	LnZ_2LnZ_4	LnZ_3LnZ_4	LnZ_1LnZ_4
	–0.09	0.06*	0.01	–0.04	–0.28*	–0.03	–0.03
	(0.11)	(0.01)	(0.07)	(0.14)	(0.14)	(0.60)	(0.04)

*Significant at 0.05 level. **Significant at 0.01 level.

Note: Asymptotic standard errors are in parenthesis.
Sample range 337…484.
Pseudo R^2=0.38.

Table 10

Derived Elasticity Estimate of Output and Supply and Demand for Variable Inputs for Mixed Cropping Belt, Punjab

	Price of Output	Price of Labour	Price of Fertiliser	Capital	Livestock	Land	Education
Derived from Parameter Estimates Translog Profit Function							
Output	0.63	–0.48	–0.53	0.57	0.14	0.65	0.01
Labour	1.25	–0.41	–0.31	0.62	0.24	0.10	0.24
Fertiliser	1.41	–0.35	–1.30	0.65	0.15	0.69	0.21
Derived from Parameter Estimates Cobb-Douglas Profit Function							
Output	1.05	–0.09	–0.97	0.24	0.14	0.37	0.24
Labour	2.05	–0.09	–1.97	0.24	0.14	0.37	0.24
Fertiliser	2.05	–1.09	–0.97	0.24	0.14	0.37	0.24

Source: Derived from Survey Data.

SUMMARY AND CONCLUSIONS

The main objective of this study was to obtain valid output supply and input demand elasticities by applying a reliable method of estimation and to, thereby, explore empirically the effects of changes in input-output prices on output and employment. These estimates have been derived from a survey data of 484 irrigated farms scattered in rice, cotton and mixed cropping zones of Punjab. The input demand and output supply elasticities have been derived by applying the normalised translog profit function to each and all zones together.

The test of the profit maximisation behaviour of the sample farmers established the validity of the translog profit function and its superiority on the Cobb-Douglas function production applied in all levels of analysis. All in all, this study has shown statistically that farmers maximise profit from farming subject to given input and output prices and quantities of fixed factors of production. As to the importance of the elasticities derived, most of them carried theoretically compatible signs and sizes. Specifically, the elasticities of labour and fertiliser with respect to the output price were generally greater than unity with many of them statistically significant. The cross-price elasticities of these variables were negative indicating them as complementary inputs. What follows from the complementarity of their relationship and statistical significance is that increasing fertiliser use increase both output supply and on-farm labour employment. Labour, in its turn, has negative own-price elasticity of demand, as expected, and greater than unity in absolute values showing it to be elastically supplied. However, the demand for labour is positively affected by increase of farm land area, capital and education. By zones, rice farmers were more responsive than their counterparts from other zones use of fertiliser and labour when output price changed. What these facts tell is that favourable fertiliser price and labour wage rate policies are conducive for generation of on-farm employment opportunities. It is more particularly so with fertiliser whose increased application under a favourable price regime can increase farm employment through additional crop husbandry and increased produce handling. Presence of other needed factor endowment facilities like capital, farmer education, etc. further increases employment of labour in agriculture.

The elasticities of output with respect to the fixed factors are positive. What this means is that since increases in all fixed factors increase the effectiveness, efficiency and marginal productivity of labour, fertiliser and other inputs, there is a need to increase their availability to induce expansion in demand for variable inputs.

This analysis concludes that farmers are price-responsive and assured prevalence of fair output and input prices is essential for preservation of farmer incentive for higher production. Further, increased fertiliser application affects on-farm employment positively. Household education increases output via increased application of modern technologies and ultimately increases on-farm employment.

POLICY IMPLICATIONS

Farmer responsiveness to farm input-output prices requires the government to ensure that the support prices announced for farm commodities preserve farmer incentive for higher production. Farm output prices need to increase overtime at least consistent with the input prices otherwise use of technological innovations like fertiliser and employment of labour are likely to be adversely affected. Since increased application of fertiliser depends on ample supply of irrigation water, the government needs to ensure the availability of adequate supply of irrigation water, which will create additional on-farm employment. To increase labour employment in agriculture further, the government should remove imperfections in rural wage formation.

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Comments

The authors need to be congratulated on presenting a paper of utmost significance which is a guide source for the policy-makers. It is a good effort since it is not easy to work with cross-sectional data while estimating input demand and supply elasticities using profit function approach. However, the paper presented by the authors is quite different than the paper given to me for comments and circulated to the audience. It was better if the authors have given me the revised paper for more thorough comments/suggestions. Anyhow, I would try to comment on both the versions that what I have read and what the have presented in the conference.

1. The sections of "Generalisation of Normalised Restricted Translog Profit Function (RTLPF)" and "Estimation Procedure" may be combined starting from the empirical model—RTLPF and share equations in the paper and the rest may be build on that. In these sections some of the material has come from Mckay, Lawrence and Vlastuin (1983), which has not been properly quoted.
2. In terms of estimation procedure, it is well established that such type of models are estimated using Zellner Efficient Estimation technique (SUR) and Three Stage Least Squares (TSLS). In the first paper only SUR was used while in the second paper both the procedures have been used. Here my concern is that the authors should directly mention the same rather than going into some other details. For the purpose of hypothesis testing the authors used the F-test at the first instance that was not the proper way of doing it. They should have used the likelihood ratio test (LR). However, they have made the correction in the paper they presented by using the LR statistics.
3. There are certain omissions or missing explanations. One of them is that of the omission of parameter restrictions: (1) profit function is of homogeneous degree one; and (2) parameter symmetry. The fact is that the empirical model they have written is actually the result of these restrictions. All the restrictions should have properly been mentioned and explained. So, the readers could have the idea that how the parameters were recovered which were not actually estimated in the model.
4. There appears to be some problems in Table 2 of the paper circulated, since the text does not match the table. I expect that the authors might have taken care of this confusion in the presented version.
5. The paper uses single output profit function, where profit and prices are normalised by output price. It is not clear which output has been used. It

needs clarification, if it is aggregate farm output then question arises how it has been constructed. How do they come up with the single output price? I think there was no need of doing it because one could always specify multi-output profit function.

6. The results of the paper, especially regarding the own price elasticity of output (next as ε_{yp}), are confusing. They estimated four models—one for the whole data and one each for the three crop ecological zones. For the whole Punjab, ε_{yp} is interpreted as own price elasticity of output that is meant for crop sector as a whole. When they estimate the model for different ecological zones, then ε_{yp} is interpreted and explained as commodity specific elasticities. No justification has been given in this regard. In my opinion they should have used multi-output profit function at the first instance. Given that what they have done, than a different model should have been specified and estimated for each commodity. I would say that as such the results are biased and should carefully be interpreted and used for further policy implications.
7. Forgetting about what the output, price and profit has been used in the analysis, dummy variables for different ecological zones should have been included in the model for Punjab as a whole to capture the weather effects, land quality, etc. Then the right way was to statistically test the zone specific effects.
8. The authors mentioned in the paper that most of the parameter estimates of the model are statistically significant. However, as I see, the results show an opposite picture.

This could be due to small variation in output and input prices.

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