Some Refinements of the Semi-Input-Output Method

by

J. TINBERGEN*

The semi-input-output method has been proposed by the author as a substitute for what was formerly called the estimation of the indirect effects of investment projects. Essentially it rests on *assumption I* that a country must aim at an "ideal development process", meaning a growth process which at no time shows unutilized production capacity. Furthermore, the method rests on *assumption II* that a distinction can be made between national or domestic activities (industries in the widest sense or sectors) on the one hand and international activities on the other hand. By definition the products of the former cannot, for technological or cultural reasons, be imported or exported. Examples have been given elsewhere [1]. Together with assumption I it follows from II that the productive capacity in the national sectors equals the demand for their products. With a given development of national income the growth of the national sectors is practically determined. The choice of new sectors to be developed must mainly be made among the international sectors, where those showing the highest comparative advantages must be chosen, if with given sacrifices of scarce factors a maximum of result is wanted. The criterion for selection of sectors will depend on the aims of policy as well as on the relative scarcity of factors. Its precise content is independent of the subject to be discussed here. For simplicity's sake we will assume that the criterion is to prefer sectors with the lowest capital-output ratio. This choice only effects the later portions of this article. The emphasis here is on something else, namely that it is not possible, if we accept assumption I, to add to the equipment of the nation a factory or set of factories in one international sector only, but that simultaneously with such an "international" investment a number of complementary investments in domestic sectors are necessary, in order to keep all capacities fully utilized. We will call the necessary combination of investments a bunch; it always consists of an investment in one international sector combined with a number of investments in all national sectors. The problem which the semi-input-output method seeks to solve is to find the composition of the bunch, expressed in terms of the additional production capacity v^h in international sector h envisaged. Our examples will take $h = 1$; in order to simplify our formulae we will assume that there are 6 international and 3 national industries. The reader

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*The author is Professor in Development~~ ~gramming at the Netherlands School of Economics and a member of the Advisory Boa· · of the Pakistan Institute of Development Economics.

will be able to generalize the formulae found for any number of each type of sector. Our example implies that no increase in capacity will take place in Sectors 2 through 6. Investments in these sectors are the alternative cases to be considered.

Assumption III is that the relationships between the production of different sectors are given by the input-output assumptions to be written as follows:

$$
\left.
\begin{aligned}
v^1 &= a^{11}v^1 + a^{12}v^2 + \ldots + a^{16}v^6 + a^{17}v^7 + a^{18}v^8 + a^{19}v^9 + c^1 + e^1 + j^1 \\
&\qquad \ldots\ldots\ldots\ldots\ldots \\
v^6 &= a^{61}v^1 + a^{62}v^2 + \ldots\ldots\ldots \quad + a^{67}v^7 + a^{68}v^8 + a^{69}v^9 + c^6 + e^6 + j^6 \\
v^7 &= a^{71}v^1 + \ldots\ldots\ldots\ldots + a^{77}v^7 + a^{78}v^8 + a^{79}v^9 + c^7 + j^7 \\
v^8 &= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
v^9 &= a^{91}v^1 + \ldots\ldots\ldots + a^{97}v^7 + a^{98}v^8 + a^{99}v^9 + c^9 + j^9
\end{aligned}
\right\} (1)
$$

Here, all symbols with the exception of the a's, which are technical coefficients, refer to changes between time 0 and time 1; their meaning is:

v^h change in volume of production of sector h

c^h change in volume of consumption of product h

e^h change in export surplus of product h

j^h change in investments taking the form of good h.

We add the definition of the change y in national income

$$
\Sigma\, y = \underset{h}{\Sigma}\, (1 - \Sigma a^{h'h})\ v^h = a^{oh}v^h \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)
$$

Everywhere prices have been taken equal to 1 and are supposed therefore not to change under any of the variations we are going to carry out.

Furthermore, we assume linear Engel functions for consumption:

$$
c^h = q^h y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)
$$

It is characteristic for the national sectors that the terms e^h do not occur in their balance equation (1).

Our solution of the problem to find the composition of the bunch of which an increase v^1 in the production of Sector 1 takes place will further be based on an *assumption IV*, that for some time these increases will be constant, meaning a linear development over time. We may call this the *Sandee* assumption since Sandee made a very efficient use of it [2]. Evidently, it can again only be on approximation.

Our problem can be defined somewhat more precisely by stating the unknowns. For the national sectors these are the v's, in our example v^7, v^8, v^9;

for the international sectors the v's are given, however; here the unknowns are the e's, which for the national sectors are given and equal to zero.

For the solution of our problem the last three equations of (1) are the hard core. In them, $v^1 \ldots \ldots v^6$ are known; the c^7, c^8 and c^9 can be expressed with the aid of (3) and (2) in terms of all v's. The only question that remains is what the values of the j^h ($h = 7, 8, 9$) are.

The most general treatment under the assumptions already made is given first; two consecutive simplifications for which further assumptions are necessary will be presented afterwards.

Solution A: In a general way we have:

$$j^h = \bar{j}^h_1 - \bar{j}^h_0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(4)$$

where the lower indices refer to time and the bars indicate absolute values. We consider all variables at time 0 given. For $\bar{j}^h_1$ we use a set of formulae:

$$\bar{j}^{hh'} = \bar{i}^{hh'} v^{h'} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(5)$$

known as the second set of input-output relations : they express that the increase $v^{h'}$ in production of commodity h' requires investment inputs of, in principle, all goods h. It is characteristic that the coefficients bear the two indices h and h' meaning that the commodity composition of the investments needed in industry h' may differ from that for another industry h''.

Going back to our example we may now rewrite the last three equations for (1), taking already into account that $v^2 = \ldots\ldots v^6 = 0$:

$$v^7 = a^{71}v^1 + a^{77}v^7 + a^{78}v^8 + a^{79}v^9 + q^7y + i^{71}v^1 + i^{77}v^7 + i^{78}v^8 + i^{79}v^9$$
$$- j^7_0$$

$$v^8 = a^{81}v^1 + a^{87}v^7 + a^{88}v^8 + a^{89}v^9 + q^8y + i^{81}v^1 + i^{87}v^7 + i^{88}v^8 + i^{89}v^9$$
$$- j^8_0$$

$$v^9 = a^{91}v^1 + a^{97}v^7 + a^{98}v^8 + a^{99}v^9 + q^9y + i^{91}v^1 + i^{97}v^7 + i^{98}v^8 + i^{99}v^9$$
$$- j^9_0$$

Filling out the expressions for y we obtain three equations in v^7, v^8 and v^9 which contain as given terms those in v^1 and the negative j^h_0 terms. Ordered in

In order to find the optimal choice we can then restrict ourselves to comparing the values of y for each case; these are equal to

$$a^{01}v^1 + a^{07}v^7 + a^{08}v^8 + a^{09}v^9 \quad\text{................................(8)}$$

for the choice of international sector 1, as in our examples. What this expression stands for is the total income increase in all national sectors plus the one in the one international sector considered. Among these are the income increases in the sectors producing investment goods. If there is — following assumption V —a constant commodity composition of investments, and if total investments are taken equal in all alternatives, then the incomes from investment sectors appearing in (8) will always be the same. It is the incomes from the other sectors which will vary and the order of attractiveness of the various bunches can be based just as well on the income increases excluding those from investment goods sectors as it can be based on the complete expression (8). This means that omitting the j^h terms from our equations (7) will not falsify our choice, to be sure, only if the conditions enumerated are all fulfilled.

REFERENCES

1. G. Rasul, *Input-Output Relationships in Pakistan, 1954.* (Rotterdam: 1964).

2. J. Sandee, *A Demonstration Planning Model for India.* (London: 1960).